INVESTIGATIONS OF THE AXISYMMETRICAL CONFINED JET

by

Wen-shiuh Hwang

B.S., National Taiwan University, Formosa, 1963

A THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1967

Approved by:

Richard M. Haynie
Major Professor

TABLE OF CONTENTS

	Page
INTRODUCTION	1
THE PURPOSE OF THE STUDY	2
SCOPE OF THE STUDY	2
THEORY	
Jets In the Infinite Field	3
Jets In the Finite Field	6
REVIEW OF LITERATURE	8
EXPERIMENTAL PROCEDURE	
Experimental Apparatus	10
Preliminary Experiments	13
Experiments For Obtaining Data	13
DATA ANALYSIS	
Method Of Analysis	15
Computation	17
DISCUSSION OF RESULTS	
Velocity Profiles	18
Maximum Velocity Along the Central Axis	24
The Velocity Phenomena In Downstream	27
CONCLUSIONS	38
RECOMMENDATIONS FOR FURTHER STUDY	39
ACKNOWLEDGMENTS	40
BIBLIOGRAPHY	41
APPENDICES	
Appendix I	
Table Of Nomenclatures	43
Appendix II	
Flow Diagrams For Digital Computer Program	45
Appendix III	
Computer Programs Used For This Study	50
Appendix IV	
Input Data to Digital Computer Program	57
Appendix V	
Output Data From Digital Computer Program	65

INTRODUCTION

The applications of the jet have been widely introduced in various engineering fields. For example, the jet water pump is one of them used in some Civil Engineering works such as deep-well pumping, booster pumping, dredging, tail water pumping and priming devices.

A jet boundary occurs between two streams which move at different speeds in the same general direction. Such a surface of discontinuity in the velocity of flow is unstable and gives rise to a zone of turbulent mixing downstream of the point where the two streams first meet. Some phenomena such as the width of the mixing region and the eddies existing in a jet are very important and of interest in practical applications. These phenomena are influenced by the velocity difference of the two streams.

As far as most engineers are concerned, high-velocity flow from a submerged outlet represents merely an irrecoverable loss of power, for a basic axiom of hydraulics states that the entire kinetic energy of such a jet will be dissipated through reactions with the surrounding fluid. The eddies generated by the velocity discontinuity between the jet and the surrounding fluid, will immediatly result in a lateral mixing process which will proceed both inward and outward with distance from the efflux section. Such lateral mixing produces a necessarily balanced action and reaction.

Literature on the problems of jets has been in existence since about 1920, but few articles have been concerned with the confined jet. However, past studies of the jet in an infinite fluid provide considerable knowledge for this special type of jet. In general, the analysis of the mixing region in a jet is very complicated and the characteristics of the confined jet have not yet been fully established.

THE PURPOSE OF THIS STUDY

The purpose of this study is to investigate the mixing process and the phenomena existing in the region of flow establishment of the confined jet. By utilizing dimensionless parameters, the mixing phenomena can be analyzed and a comparison may be made with the case of the jet in the infinite field.

SCOPE OF THE STUDY

The experiments were conducted in this way; the jet was formed by issuing a flow from a 5/8" diameter steel pipe into a secondary flow which was flowing in the same direction in a 5" diameter plexiglas pipe. The jet was discharged along the central axis of the main pipe. The confined jet was axisymmetrical.

The velocities of both the jet and the secondary flow were varied from 0 to 16.269 ft/sec. for the jet and from 0 to 5.277 ft/sec. for the secondary flow. The total quantity of the discharge of these two streams varied from 0.014 to 0.721 cfs. Four series of runs were used for obtaining the data for this study. Each series consisted of a constant secondary flow with five or six runs of different velocities, from 0 to 16.269 ft/sec., for the jet.

In order to determine the magnitude of the velocity changes in the main pipe, the coordinate distribution method was used. The velocities for each run were measured at intervals of 1/2" along the vertical diameter of the cross section and each 4" along the pipe past the outlet of the jet.

THEORY

Since it will be helpful for the further study of the confined jet, it is desirable to introduce some knowledge useful in the analysis for the case of a jet in an infinite field. The dimensional aspects of the phenomenon are first discussed, followed by the elementary physical analysis of the mean flow pattern.

Jets In the Infinite Field

If the Reynolds number for the fluid efflux from a submerged boundary outlet is not too low, the mean velocity V at any point should depend only on the coordinates X, Y, Z, on the efflux velocity V_o, and on a linear dimension L_o characterizing the particular outlet form.

The dimensionless relationship of these variables may be grouped as

$$\frac{V}{V_o} = f_1 \left(\frac{X}{L_o}, \frac{Y}{X}, \frac{Z}{X}\right) \text{---------------(1)}$$

The differential equation of continuity is

$$\frac{\partial V_x}{\partial X} + \frac{\partial V_y}{\partial Y} + \frac{\partial V_z}{\partial Z} = 0 \text{----------------(2)}$$

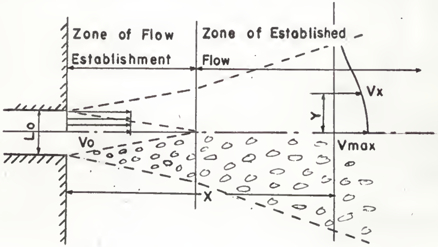


Figure 1. Schematic Representation of Jet Diffusion

The rate of flow Q, the momentum flux M, and the energy flux E past successive normal sections may be respectively written as the integral of the differential flux $V_x dA$, $(\rho V_x)V_x dA$ and $(\rho V^2/2)V_x dA$ over any normal section.

Therefore, the folling relations can be obtained

$$\frac{Q}{Q_o} = \frac{\int_0^\infty V_x dA}{V_o A_o} = f_2 \left(\frac{X}{L_o}\right) \text{-----------(3)}$$

$$\frac{M}{M_o} = \frac{\int_0^\infty (V_x)^2 dA}{(V_o)^2 A_o} = f_3\left(\frac{X}{L_o}\right) \text{--------------} (4)$$

$$\frac{E}{E_o} = \frac{\int_0^\infty V^2 V_x dA}{(V_o)^3 A_o} = f_4\left(\frac{X}{L_o}\right) \text{--------------} (5)$$

Where the subscript "o" indicates the quantity at the efflux cross section and the non subscripted quantity is any arbitrary section downstream of the efflux cross section.

For the case of the jet in the infinite field, the tangential shear force within the mixing region should decelerate the jet and accelerate the surrounding fluid. Since this process is wholly internal, it follows that the momentum flux must be a constant for all normal sections of a given flow pattern.

$$\frac{M}{M_o} = \frac{\int_0^\infty (V_x)^2 dA}{(V_o)^2 A_o} = 1 \text{--------------------} (6)$$

Experimental data follow the general trend of the Gaussian normal probability function which is

$$\frac{V_x}{V_{max}} = \exp\left(-\frac{Y^2}{2\sigma^2}\right) \text{----------------------} (7)$$

where V_x, V_{max}, Y and σ are as shown in Figure 2.

The use of eq. 7 in the analysis permits characteristics of the entire flow pattern to be expressed in terms of the two parameters which define the proportions of the curve; the velocity V_{max} and the standard deviation σ.

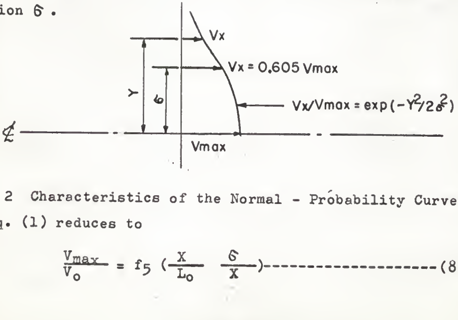


Figure 2 Characteristics of the Normal - Probability Curve

Then Eq. (1) reduces to

$$\frac{V_{max}}{V_o} = f_5\left(\frac{X}{L_o} \quad \frac{\sigma}{X}\right) \text{-------------------} (8)$$

The condition of dynamic similarity simultaneously requires that at any cross section, regardless of the efflux velocity

$$\frac{\sigma}{X} = C \text{ --------------------------------(9)}$$

In other words, the angle of jet diffusion must be constant.

Based on the above general consideration, we make an assumption for the two-dimensional case such that

$$\frac{\sigma}{X} = C_1 \text{ --------------------------------(10)}$$

Equation (6) of the momentum flux will lead to the expression

$$\frac{V_{max}}{V_o} = \sqrt{\frac{1}{\sqrt{\pi}\, C_1} \frac{B_o}{X}} \text{ -------------------(11)}$$

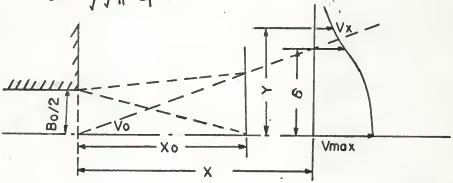


Figure 3 Definition Sketch for Zone of Flow Establishment

The distribution of the longitudinal velocity component in the zone of the established flow may then be written as

$$\frac{V_x}{V_o} = \sqrt{\frac{1}{\sqrt{\pi}\, C_1} \frac{B_o}{X}} \quad \operatorname{Exp}\left[- \frac{1}{2(C_1)^2} \frac{Y^2}{X^2}\right] \text{(12)}$$

$$\frac{Q}{Q_o} = \sqrt{2\sqrt{\pi}\, C_1 \frac{X}{B_o}} \text{ ------------------(13)}$$

$$\frac{E}{E_o} = \sqrt{\frac{2}{3\sqrt{\pi}\, C_1} \frac{B_o}{X}} \text{ ----------------(14)}$$

For the three-dimensional case, we can replace Y and B_o by R and D_o and integrate over the corresponding areas. Then the equivalent expressions for the condition that $\sigma/X = C_2$ are

$$\frac{V_{max}}{V_o} = \frac{1}{2C_2} \frac{D_o}{X} \text{ --------------------(15)}$$

$$\frac{V_x}{V_o} = \frac{1}{2C_2} \frac{D_o}{X} \exp\left[-\frac{1}{2(C_2)^2} \frac{R^2}{X^2}\right] \text{-----(16)}$$

$$\frac{Q}{Q_o} = 4C_2 \frac{X}{D_o} \text{------------------------------(17)}$$

$$\frac{E}{E_o} = \frac{1}{3C_2} \frac{D_o}{X} \text{------------------------------(18)}$$

Under this analysis, Hunter Rouse (6) performed a very successful experiment in 1948 and obtained the following results;

For the two-dimensional case

$$C_1 = 0.109 \text{--------------------------------------(19)}$$

$$\frac{V_{max}}{V_o} \sqrt{\frac{X}{B_o}} = 2.28 \text{----------------------------(20)}$$

$$\frac{Q}{Q_o} = 0.62 \sqrt{\frac{X}{B_o}} \text{---------------------------(21)}$$

$$\frac{E}{E_o} = 1.86 \sqrt{\frac{B_o}{X}} \text{---------------------------(22)}$$

For the three-dimensional case

$$C_2 = 0.081$$

$$\frac{V_{max}}{V_o} \frac{X}{D_o} = 6.2 \text{-------------------------------(23)}$$

$$\frac{Q}{Q_o} = 0.32 \frac{X}{D_o} \text{-------------------------------(24)}$$

$$\frac{E}{E_o} = 4.1 \frac{D_o}{X} \text{-------------------------------(25)}$$

Jets In the Finite Field

The primary assumption made for the jet in the infinite field was the constancy of momentum flux past successive sections. The condition of continuity was then satisfied by lateral flow from or to infinity. If the field of flow is finite, such lateral flow cannot occur at the outer limits. The discharge, instead of the momentum flux, past successive transverse sections must then be constant. The resultant change in momentum from section to section of the zone of diffusion requires that there be an accompanying pressure gradient.

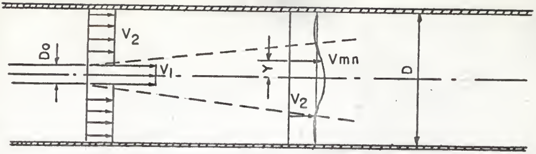


Figure 4 Flow from a Jet in the Bounded Region

For a jet in a bounded region, as shown in Figure 4 the outer limit of the zone of diffusion will be that radius at which the velocity is equal to V_2.

Although, from the experimental data (see Figure 9 to 13), the velocity profile in the central part of the conduit still has the property of the Gaussian normal probability function,

$$\frac{U_{mn}}{U_{max}} = \exp\left(\frac{-Y^2}{2\sigma^2}\right) \text{------------------------} (26)$$

the value of σ/X will not be a constant but will be determined by the effects of the pressure gradient. Since σ/X is not a function of the Gaussian normal probability function, the analysis used for the previous case of the free jet is not applicable.

However, the general consideration of Eq. (1), (2), (3), (4), and (5) are still satisfied if we replace all velocities by the relative velocities

$$U_{mn} = V_{mn} - V_2, \; U_{max} = V_{max} - V_2, \; U_o = V_1 - V_2 \text{-------} (27)$$

and make the range of the integration extend from 0 to D/2.

By analogy to the case of the mixing of streams, one would expect that the jet would diffuse at smaller and smaller angles as the ratio of the velocities $(V_1 - V_2)/V_1$ vary from 0 to 1. At the limit $V_1 = V_2$, the rate of spread resulting from shear-generated turbulence would be zero.

The discharge $\int_0^{D/2} (V_x - V_2) 2\pi R dR$ past successive sections of the zone of diffusion must increase as the width of the zone increases. For an infinite field the increase in discharge would be supplied by

lateral inflow. For a finite field, if the ratio D/D_o is large but finite, an increase in jet discharge is characterized by a slight reduction of the velocity outside the zone of diffussion. If V_2 is equal to zero, any reduction in the outside velocity obviously means a reverse flow from the diffusion jet to the area adjacent to the nozzle.

From the above discussion, the assumption of similarity must be retained also, but it may well require a different form than is used for the infinite field. Some dimensionless group relationships can be used to describe the above discussed phenomena.

REVIEW OF LITERATURE

In 1934, Kuethe (7) investigated the turbulent mixing regions formed by a plane jet and an axially symmetrical jet issuing into a fluid at rest and he also extended his investigation to consider the general case of the mixing of two parallel streams of different velocities. Velocity profiles in the mixing region of an axially symmetrical jet were measured by means of a pitot tube and good agreement was found between theory and experiment.

In 1948, Rouse (6) made some assumptions when he studied the diffusion of submerged jets and obtained analytical relationships. The measurements were shown to be in substantial agreement with the analytical relationships. He also provided the single experimental coefficient required to complete the analysis in the two-dimensional and in the three-dimensional cases. All results were reduced to a form immediately useful for design purposes for the flow of any liquid or gas at moderate to high Reynolds numbers.

In 1955, Weinstein, Osterle, and Forstall (8) studied the momentum diffusion from a slot jet into a moving secondary fluid. By introducing the correlation theory they concluded that the spreading coefficients for round and slot jets are independent of axial distances. This is analogous to results obtained for the case of a particle stream issuing from a point or line source into a field of homogeneous isotropic turbulence. Comparisons made between the slot jet and round jet diffusion processes indicate a relationship between the two mixing phenomena.

In 1964, Barchilon and Curtel (12), based on Craya's "Approximate confined jet theory" which is based on the Reynolds number and the continuity equations, studied the details of the structure of an axisymmetric confined jet with backflow. They used two experimental apparatuses in the investigation; one utilized water and the other air. The experimental results have made it possible to characterize the mean structure of the eddy in terms of the similitude parameter Ct.

In 1964, Mueller (13) published a paper dealing with the determination of the optimum dimensions of the water jet pump so that the best efficiency could be obtained. He introduced several water jet pumps used in civil engineering such as indicated above in the introduction. Recommendations for the design of water jet pumps were also given.

In 1962, Maczynoski (11) added the consideration of the pressure factor into his study of a round jet in an ambient co-axial stream. The theorectical analysis indicates that the spread of the profile can be described by a characteristic length scale. The experimental results showed that near the center of the jet the profiles of the flow follow the Gaussian form but that near the edge they fall off with distance somewhat more steeply than the Gaussian form and the variation of the lateral influx appears to be an important feature distinguishing jets with and without ambient streams.

EXPERIMENTAL PROCEDURE

Experimental Apparatus

A photograph of the experimental apparatus is presented in Figure 5, and the schematic diagram is shown in Figure 6. Two stilling tanks were used in this experiement; a 2'x5'x7' tank used at the upstream end and a 2½'x4'x2½' tank used at the downstream end. A 5" diameter plexiglas pipe 6' in length, which connected these two tanks, was used for the main experiment.

Water was pumped from a reservoir and two water supply lines were used to make the jet and the secondary flow. Water for the jet with a velocity from 0 to 16.269 ft/sec. was passed through a 5/8" diameter hose and a 5/8" diameter steel pipe which was placed along the central axis of the main pipe. The jet was introduced at a distance of 1' downstream from the entrance of the main pipe. Water for the secondary flow with a velocity of 0 to 5.277 ft/sec. was supplied by a 4" diameter pipe. It flowed into the upstream stilling tank and was passed through the main pipe where it joined the jet. The mixed flow then proceeded downstream to the stilling tank where the total quantity was measured at the weir.

In order to avoid the effects of the oscillation and to get a uniform secondary flow, a double-level screen was placed beneath the outlet of the 4" water supply pipe and a bellmouth of 4" in curvature was fixed to the entrance of the main pipe.

A piezometer tube fastened along the upstream tank wall was used for the upstream water level reading. On the main pipe, four pitot-static tubes (see Figure 5) were spaced at 4" intervals from the outlet of the jet. They were used for measuring the velocities in the main pipe. The scales on the manometer tubes connected to the pitot tubes were graduated in increments of 0.001 ft.

A 7 7/16" x 9 1/2" rectangular weir was placed at the outlet of the downstream tank for measuring the total discharge. The reading of the water head at the weir was given by a point gage which was graduated in increments of 0.001' and was located at 1'-11" upstream from the weir.

Fig. 5 The Pitot-static Tube Used in This Experiment

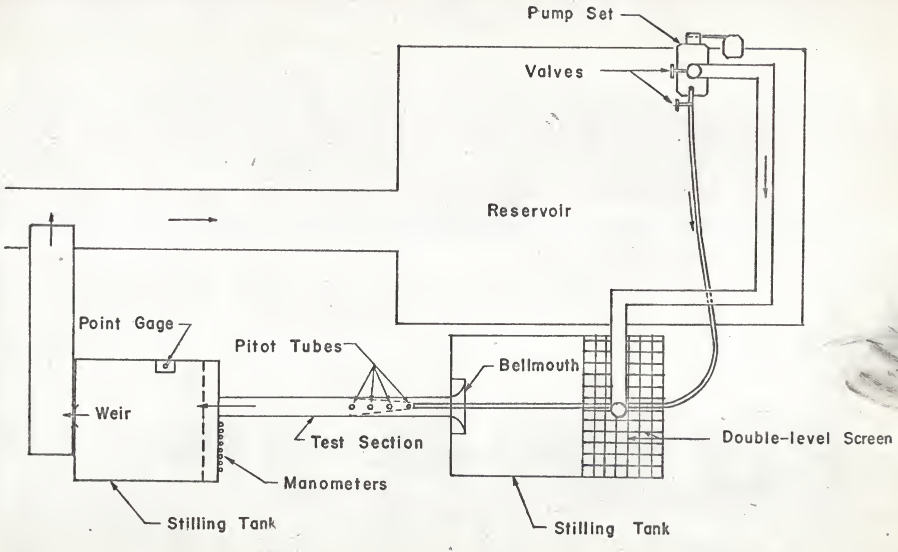


Fig. 6 Experimental Apparatus

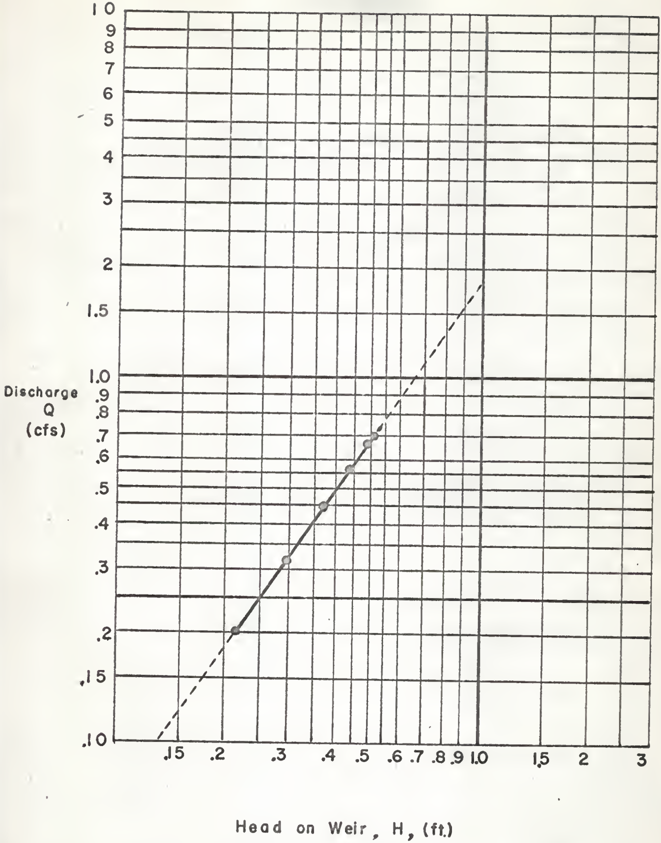


Fig. 7 The Discharge Curve at the Weir

Preliminary Experiments

There were two preliminary experimental procedures: the calibration of the weir; and, the determination of the coefficients of the pitot-static tubes. A rectangular weir was placed at the downstream reservoir outlet for determining the total discharge of the system.

The weir was calibrated over the range of flows. By timing the flow of a certain amount of water into a measuring tank and reading the corresponding head at the weir, the rate of flow could be calculated. The calibration curve is plotted as Figure 7.

After the weir was calibrated, coefficients of the pitot-static tubes were determined. The method of traversing the velocity profiles was used in this calibration. The head readings from the manometer were used to determine the velocity profile ($V_p = \sqrt{2GH}$) and the mean of the integration of this profile gave the pitot-tube-measured mean velocity. Meanwhile, the head reading at the weir gave the real mean velocity in the pipe ($V_m = Q/A$). Therefore, the coefficient of the pitot-static tube was $K_m = V_p/V_m$. Several different rates of flow were used for each pitot-static tube calibration. The calculations and the data are given in appendices III, IV, and V. The results show that the coefficients for pitot tubes 1, 2, 3, and 4, were respectivly 0.986, 0.956, 0.932, and 0.961.

Experiments For Obtaining Data

After the weir and the pitot tubes had been calibrated, the main part of the research followed. The experiments included four series of runs. The velocities of the secondary flow were kept nearly constant but the velocity of the jet was varied for each run in a series.

In the first series, the secondary flow was zero and the velocity of the jet was varied from 0 to 16.269 ft/sec. (five runs among this range). For each run, the water heads were read both at the head tank and at the downstream weir which was used to measure the quantity of flow. Meanwhile, each pitot tube was moved upward over the upper half diameter of the main pipe, and measured successively the velocities of each section at 0,

0.166, 0.333, 0.500, 1.000, 1.500, 2.000, and 2.475 inches from the central axis of the main pipe.

In the same manner, the velocity of the secondary flow was varied from 0 to 5.277 ft/sec. and the velocity of the jet was varied over the same range as above. The data collected in each run are given in Appendices IV and V.

DATA ANALYSIS

Method of Analysis

As mentioned above, the relationships among certain dimensionless groups can be used for the analysis of this experiment. The phenomena existing in the confined jet will be described and discussed.

The jet and the secondary flow are assumed to be uniform before they first meet. The experimental results (see Figures 9 to 13) show that this assumption is acceptable. All the analyses are based on the values of $(V_1 - V_2)/V_1$ from 0 to 1.

The velocity of the jet for each run was obtained from the manometer reading of the head on the first pitot-static tube which was located directly in front of the nozzle of the jet. This velocity is expressed as

$$V_1 = K_1 \sqrt{2GH_1} \qquad (28)$$

where H_1 is the velocity head reading on the pitot-static tube.

$K_1 = 0.986$, is the coefficient of the pitot-static tube.

Therefore, the rate of jet flow will be

$$Q_1 = (V_1).(A_o)$$
$$= (K_1 \sqrt{2GH_1})\ (D_o^2/4) \qquad (29)$$

where $D_o = 5/8''$, is the diameter of the jet nozzle.

The head reading at the weir and the calibration curve (Figure 6) give us the total discharge, Q, of the jet and the secondary flow. Then, the rate of the secondary flow will be

$$Q_2 = Q - Q_1 \qquad (30)$$

And the velocity of the secondary flow will be

$$V_2 = Q_2/(A - A1)$$
$$= Q_2/(D^2 - D_1^2)/4 \qquad (31)$$

where $D = 5''$, is the diameter of the main pipe.

$D_1 = 11/16''$, is the external diameter of the jet conduit.

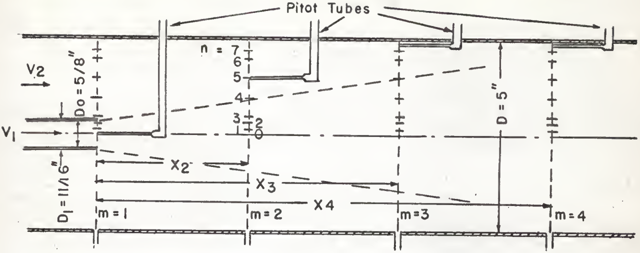


Figure 8 Zone of the velocity measurements

The velocities at the points of each section can be calculated if the manometer head readings are given (see Figure 8)

$$V_{mn} = K_m \sqrt{2GH_{mn}} \qquad (32)$$

where m= cross sections 1, 2, 3, and 4.

H_{mn}= the velocity head readings of the pitot-static tubes.

When all these velocities have been obtained, the following modified velocities can be determined;

$$U_{mn} = V_{mn} - V_2 \qquad (33)$$

$$U_{max} = V_{max} - V_2 \qquad (34)$$

$$U_o = V_1 - V_2 \qquad (35)$$

With the above values, the following dimensionless groups are obtained;

$(R-D_o/2)/X$	Vs	U_{mn}/U_o
R/X	Vs	U_{mn}/U_{max}
X/D_o	Vs	U_{max}/U_o

Computation of Dimensionless Groups

All the experimental data were punched on IBM cards. The required computations were performed on the IBM 1620 Digital Computer. The flow diagrams are shown in Appendix III. All the input data have been printed in tabular form in Appendix IV and all the output data have been printed in Appendix V.

DISCUSSION OF RESULTS

Velocity Profiles

The case of the secondary flow equal to zero was studied first. It is evident, from Figure 9 that a negative velocity existed outside of the diffusion region. This phenomenon indicates that an adverse pressure gradient was present and that circulation occurred within this region. As discussed in the theory, this adverse pressure gradient must satisfy the change in momentum requirements. Comparing the two runs in Figure 9, it is seen that the circulation grows as the velocity difference (V_1-V_2) increases. The Gaussian normal function is not able to describe the whole velocity profile for this case, because of the existance of the circulation.

For the cases with non-zero secondary flow, all the velocity profiles, plotted at the section where the two streams first meet, indicate that the velocity of two streams are uniform, and the velocity profiles downstream from this section seem to change from section to section in a very reasonable manner. The Gaussian normal function appears to satisfy the region near the longitudinal axis of the pipe. Figures 10 through 13 indicate no reduction of the outside velocity due to the effects of the jet diffusion. However, the experimental data in Appendix V show that the phenomenon of the velocity reduction is present. There is no circulation evident in any of the runs having secondary flow, but it is believed that the circulation will occur if the velocity difference becomes very large. The plotted data indicate that the distance required for the establishment of mixing over the entire diameter of the pipe is proportional to the velocity difference. After the mixing has been completed, the flow will proceed downstream with a new modified profile.

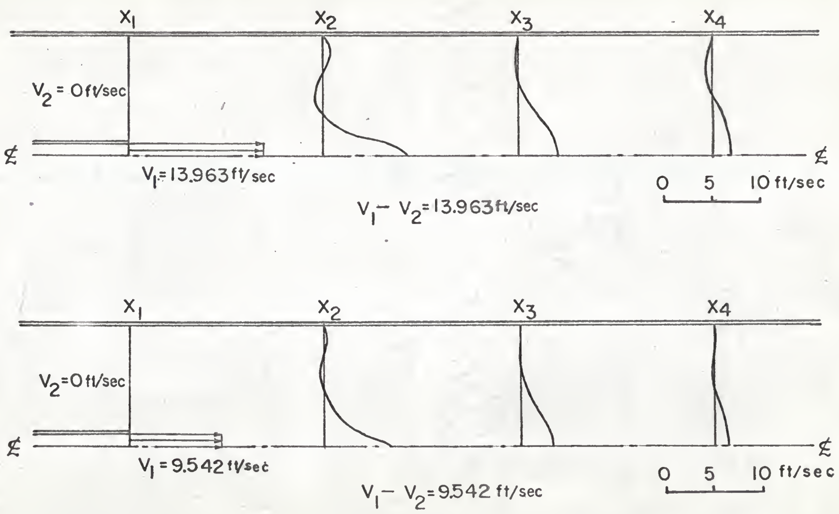


FIG. 9 VELOCITY PROFILES (A)

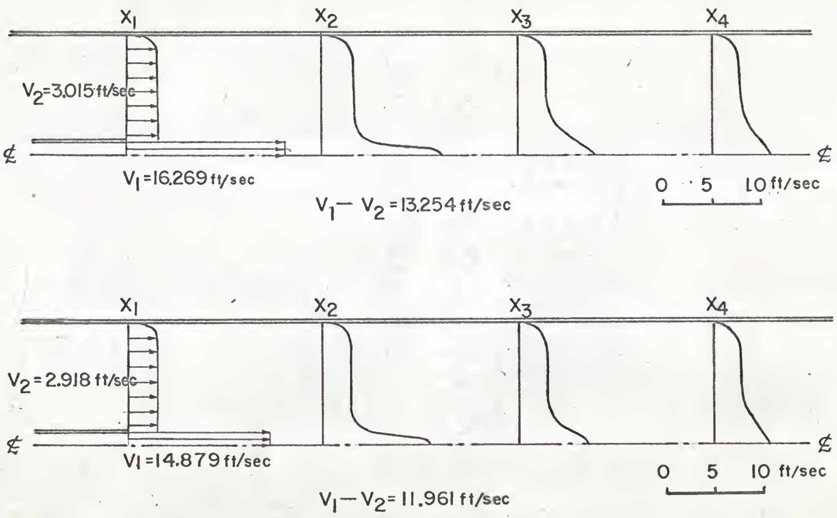


FIG. 10 VELOCITY PROFILES (B)

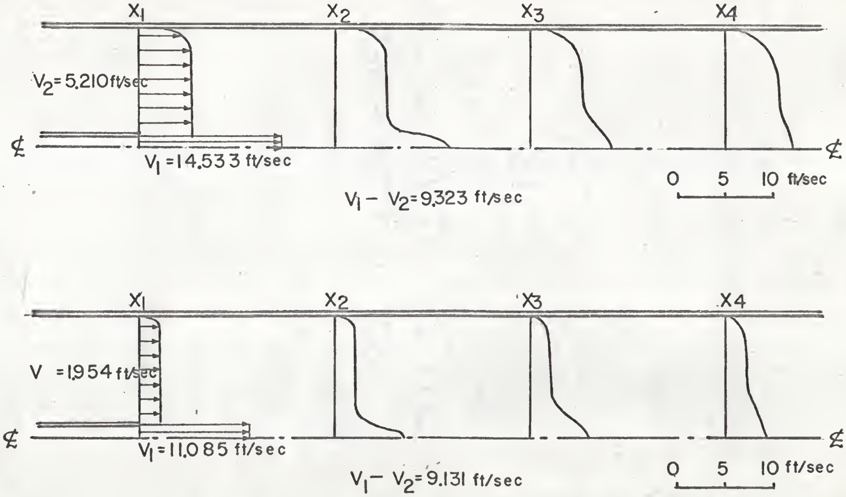


FIG. 11 VELOCITY PROFILES (C)

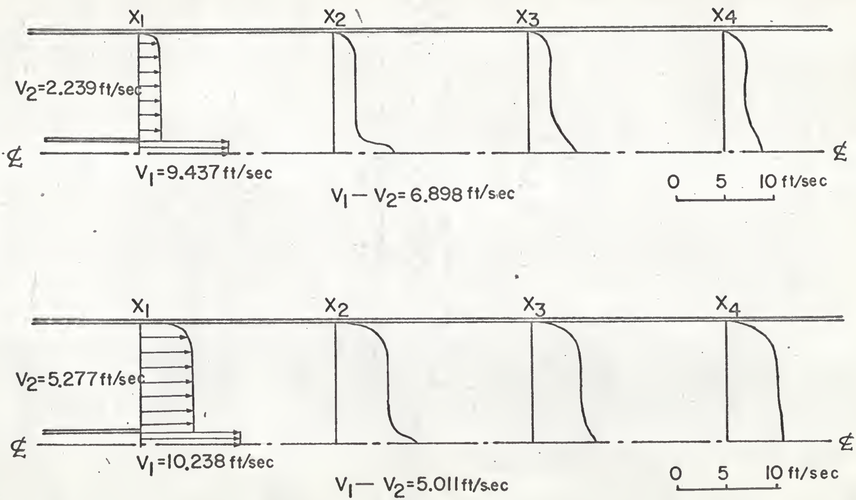


FIG. 12 VELOCITY PROFILES (D)

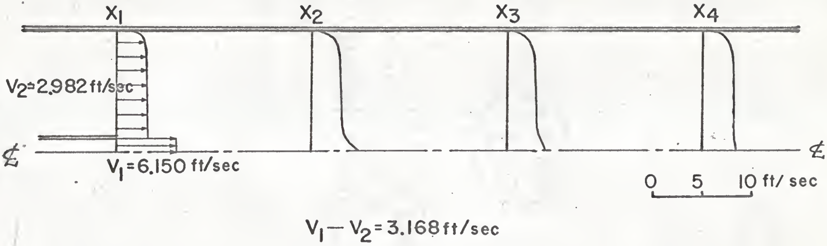


FIG. 13 VELOCITY PROFILES (E)

Maximum Velocity Along the Central Axis

From Figures 14 and 15, it is seen that the maximum excess velocity in the jet is inversely proportional to the distance from the jet nozzle. As the jet moves downstream, the value of X/D_o increases and the curve changes gradually from a linear relationship to a non-linear relationship. For the case of the jet in an infinite field, the U_{max}, based on the theoretical analysis, will approach zero as the distance X goes to infinity. For the jet in the finite field, since continuity must be maintained, the U_{max} will approach a constant after the mixing process developes over the entire pipe cross section.

Figure 15 shows that the values of X/D_o at which the values of U_{max}/U_o approach a constant is dependent on the value of $(V_1-V_2)V_1$. Obviously, as $(V_1-V_2)/V_1$ increases the value of X/D_o at which U_{max}/U_o approaches a constant also increases. The value of U_{max}/U_o for the case of $(V_1-V_2)/V_1 = 0.320$ is about 0.105. The curves in this figure suggest that all the final values of U_{max}/U_o approach this value at some value of X/D_o.

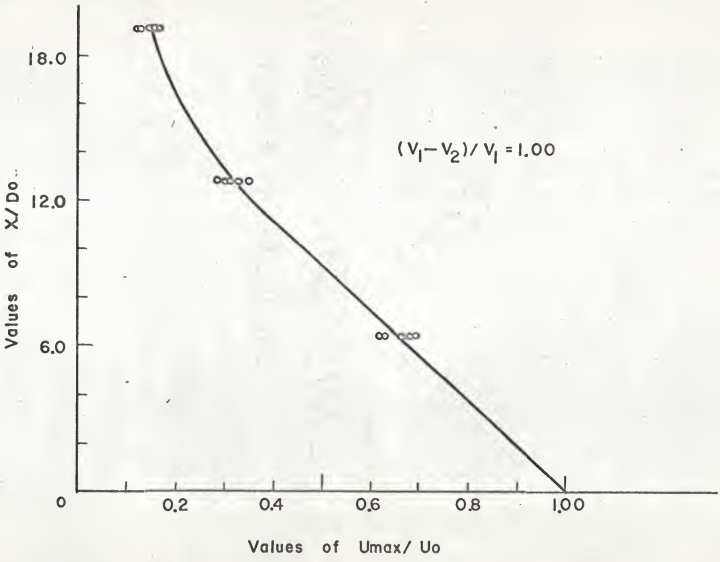


Fig. 14 Distribution of Velocity Along the Central Axis (A)

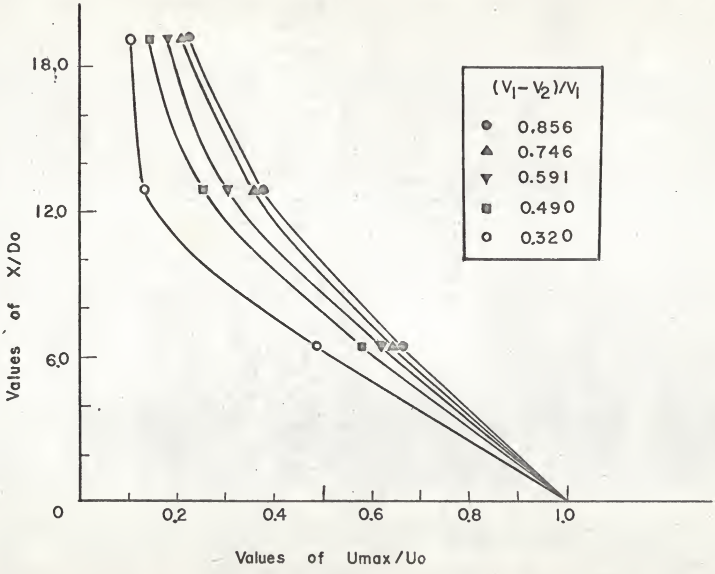


Fig. 15 Distribution of Velocity Along the Central Axis (B)

The Velocity Phenomena Downstream of the Jet

As mentioned above, the experimental results show that an adverse pressure gradient exists in the region outside the diffusion zone.

Figures 16 thru 18 show the relationship of U_{mn}/U_{max} at given cross sections of the pipe and at various ratios of R/X. The value of U_{max} will be positive for all runs. Therefore, an adverse pressure gradient exists if the value of U_{mn}/U_{max} is negative. The values of R/X at the wall of the main pipe for the section X_2, X_3, and X_4 are respectively 0.630, 0.315, and 0.210. Some negative values of U_{mn}/U_{max} existed near the wall for each of the above cross sections, but the effects are considered to be due more to adhesion to the wall than to the pressure gradient.

The values of U_{mn}/U_{max} of most of the runs were positive at section X_2 (Figure 16), since the diffusion of the jet is still narrow when it passes this section and the flow carried from the outside stream is so small that it does not create an adverse pressure gradient. This situation has been changed by the time the flow reaches section X_3. At section X_3 negative values of U_{mn}/U_{max} are present over a large range of R/X (see Figure 17). This indicates that there is an adverse pressure gradient existing in this region. When secondary flow is zero, this adverse pressure gradient will cause recirculation. As the jet moves downstream to section X_4 (Figure 18), the value of U_{mn}/U_{max} becomes positive for most runs. This change illustrates that the adverse pressure gradient has disappeared and the mixing has been completed over the entire cross section. The flow will proceed downstream with a new modified velocity profile. The effects of the jet will decrease as the flow moves downstream.

Another dimensionless group, based on the values of U_{mn}/U_o (Figure 19 thou 24), have been used to describe how the velocity profile for each run changes as the flow moves downstream. These curves show the same phenomena observed above; the largest adverse pressure gradient occurs at section X_3 and disappears at section X_4. The adverse pressure gradient exists only in the zone where mixing is present.

The value of U_{mn}/U_o is always positive as long as the value of $(R-D_o/2)/X$ is less than zero. It is found that no adverse pressure gradient exists near the central axis. At section X_3 the region of adverse pressure gradient moves nearer the central axis as the values of $(V_1-V_2)/V_1$ become smaller.

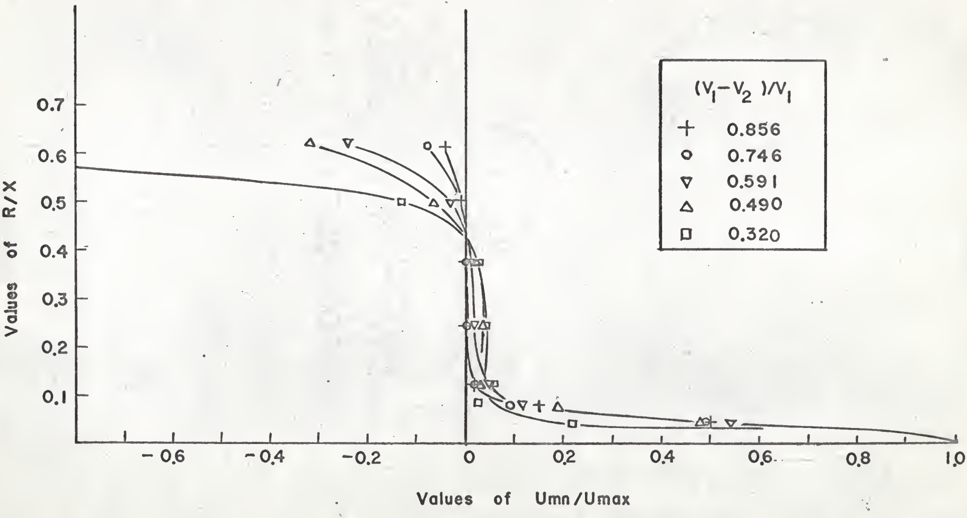


Fig. 16 Distribution of Velocity Characteristics in Flow (At X_2)

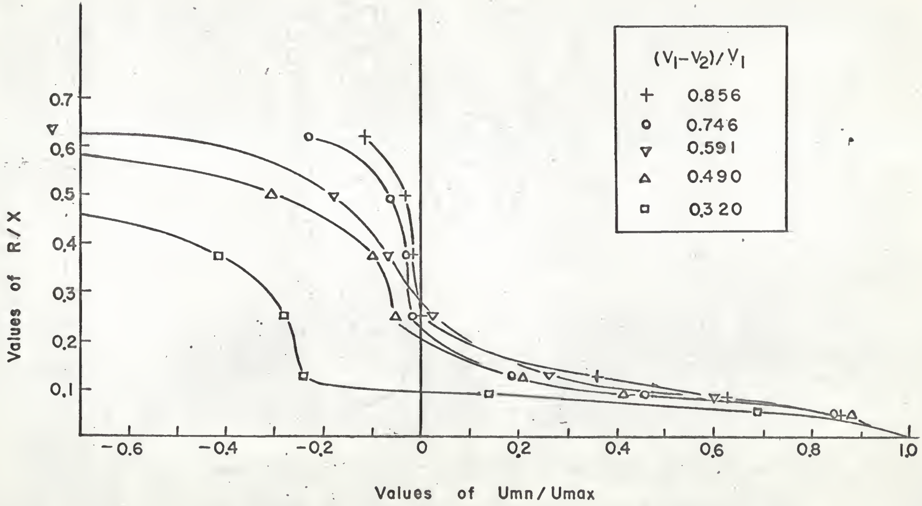


Fig. 17 Distribution of Velocity Characteristics in Flow (At X_3)

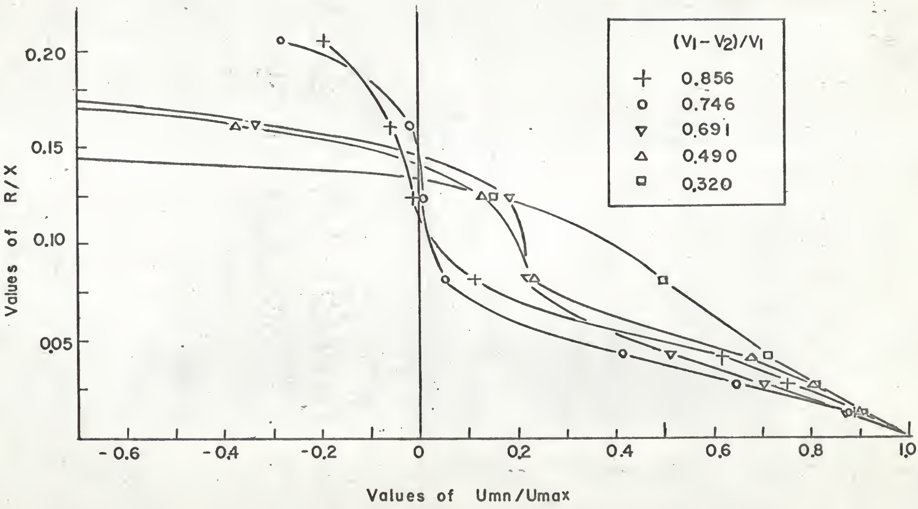


Fig. 18 Distribution of Velocity Characteristics in Flow (At X_4)

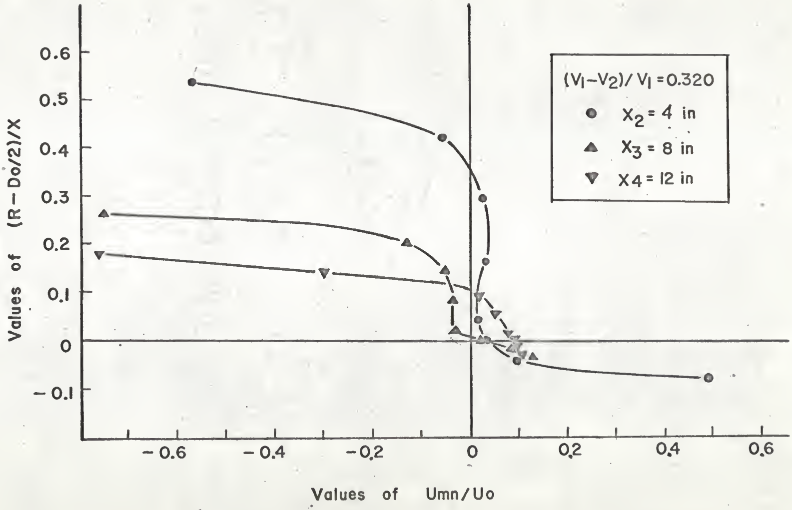


Fig. 19 Distribution of Longitudinal Velocity of Flow from Orifice (A)

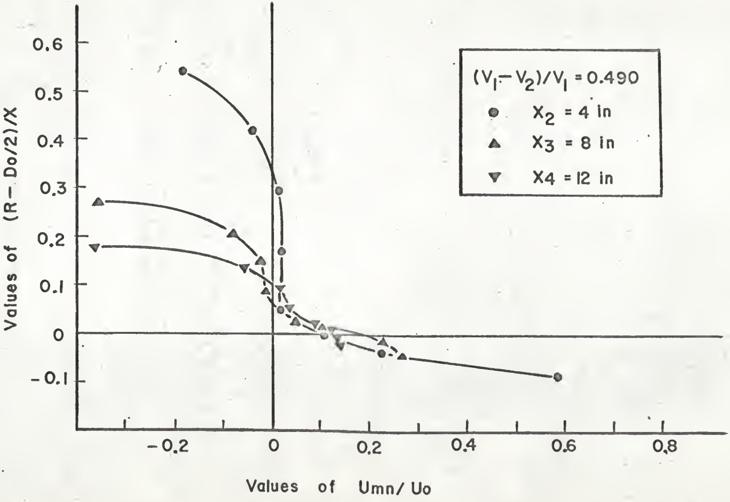


Fig 20 Distribution of Longitudinal Velocity of Flow from Orifice (.B)

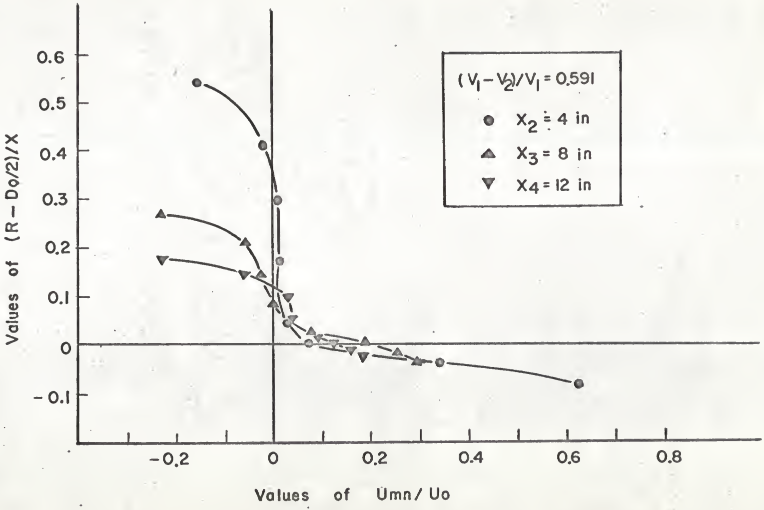


Fig. 21 Distribution of Longitudinal Velocity of Flow from Orifice (C)

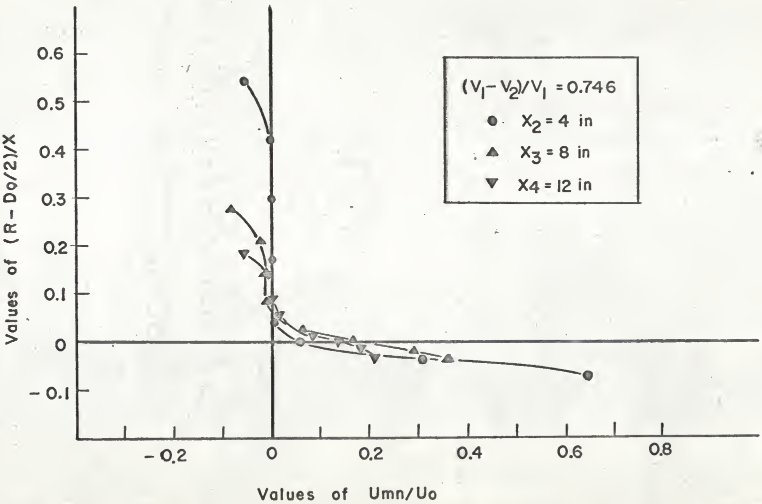


Fig. 22 Distribution of Longitudinal Velocity of Flow from Orifice (D)

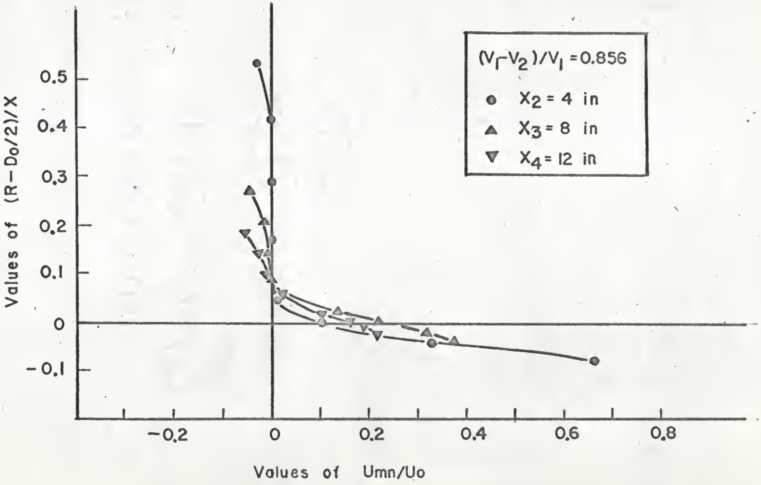


Fig 23 Distrbution of Longitudinal Velocity of Flow from Orifice (E)

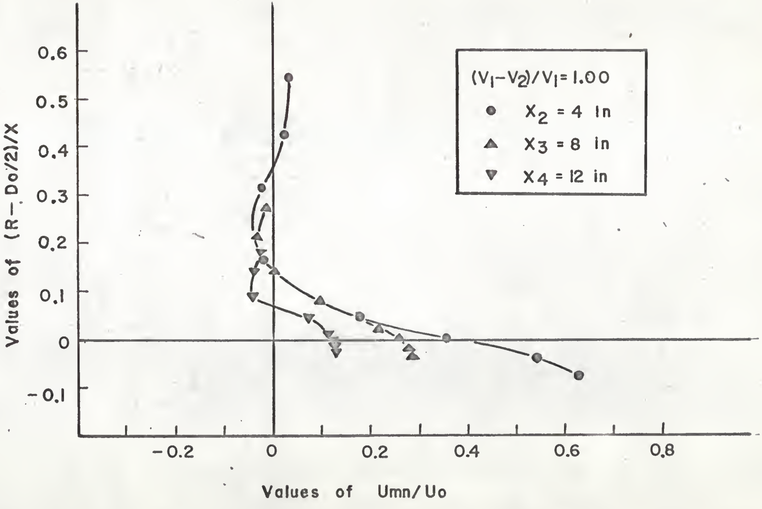


Fig. 24 Distribution of Longitudinal Velocity of Flow from Orifice (F)

CONCLUSIONS

The characteristics of the jet diffusion in the finite field are quite different from those of the jet in the infinite field. In general, the former case is more complicated.

It was found that the Gaussian function, which was used very successfully in the study of a free jet, can be used for only a small region in the central part of the velocity profile for the confined jet.

It was found that the experimental results could be analyzed by means of dimensionless parameters. The experimental results show that, in contrast to the case of the infinite field in which the mixing continues as long as there is flow and the mixing extends to infinity, the mixing of a confined jet is completed within a determinable distance after the jet and the secondary flow meet. The mixed flow will continue downstream with a fixed profile.

The fundamental theoretical difference between the case of the finite field and infinite field is that finite field flow must satisfy the condition of continuity rather than the constancy of the momentum flux which must be satisfied in the infinite field. As mentioned in the theory, the pressure gradient, which may be influenced by many factors, will play an important role in the confined jet study.

RECOMMENDATIONS FOR FURTHER STUDY

The following are recommended as subjects for further investigation.

Since the difference of the velocities of two streams will influence the production of the adverse pressure gradient, the investigation of higher velocity jets is suggested. The higher the velocity of the jet the more pronounced will be the influence. For such an investigation, a more powerful jet resource is required.

The above idea of changing the velocity leads to another suggestion. If the direction of one of the streams is changed, the effects of the adverse pressure gradient will also change. Therefore, let two streams meet from converging directions.

In general, as the ratio D/D_o decreases, the reduction of the velocity outside the diffusion region will become more pronounced and reverse flow may occur (depending on the ratio V_2/V_1). It is suggested that an investigation be made by changing the ratio D/D_o. If the ratio is near 1, the effects of the shear forces existing along the solid boundary should be considered. It may be possible to incorporate boundary layer theory in this case.

ACKNOWLEDGMENTS

The author acknowledges the counsel, guidance and critical review given by the major instructor of this study, Dr. Richard M. Haynie, Department of Civil Engineering, Kansas State University. His encouragement and efforts made possible the completion of this investigation.

The author wishes to express his appreciation for review of this thesis and the helpful suggestion contributed by Dr. Jack B. Blackburn, Head of the Civil Engineering Department, Kansas State University.

Acknowledgment is also given to Professor John E. Kipp and Dr. S. Thomas Parker for being on the advisory committee and reviewing the manuscript.

BIBLIOGRAPHY

1. Li and Lam, Principles of Fluid Mechanics, Addison-Wesley Co., 1964.
2. Hermann Schlichting, Boundary Layer Theory, McGraw-Hill Book Co., 1962.
3. Shi-I Pai, Fluid Dynamics of Jets, D. Van Nostrand Co., 1954.
4. J. O. Hinze, Turbulence, McGraw-Hill Book Co., 1959.
5. Hunter Rouse, Advanced Mechanics of Fluid, John Wiley & Sons, Inc., 1959.
6. Hunter Rouse, "Diffusion of Submerged Jets," Transactions, A.S.C.E., Vol. 74, Dec., 1948.
7. A. M. Kuethe, "Investigations of the Turbulent Mixing Region Formed by Jets," Transactions, A.S.M.E., Vol. 57, 1935.
8. A. S. Weinstein, J. F. Osterle and W. Forstall, "Momentum Diffusion From a Slot Jet Into a Moving Secondary," Journal of Applied Mechanics, Transactions, A.S.M.E., Vol. 23 n3, Sept., 1956.
9. J. A. Goff and C. H. Coogan, "Some Two-dimensional Aspects of the Ejector Problem," Transactions, A.S.M.E., Vol. 64 Dec., 1942.
10. G. K. Batchelor and A. E. Gill, "Analysis of the Stability of Axisymmetric Jets," J. Fluid Mechanics, University of Cambridge, England, Vol. 14, 1962.
11. J. F. J. Maczynski, "A Round Jet in an Ambient Coaxial Stream," J. Fluid Mechanics, University of Cambridge, England, Vol. 13, Aug., 1962.
12. M. Barchilon, "Some Details of the Structure of an Axisymmetric Confined Jet with Backflow," Journal of Basic Engineering, Paper 64, Transactions, A.S.M.E., 1964.
13. N. N. G. Mueller, "Water Jet Pump," Proceedings, A.S.C.E., Vol. 90, No. HY3, May , 1964.
14. G. Abraham, "Horizontal Jets in Stagnation Fluid of Other Density," Proceedings, A.S.C.E., Vol. 91, No. HY4, July, 1965.

APPENDICES

APPENDIX I

TABLE OF NOMENCLATURE

A	:	cross sectional area of the main pipe
A_o	:	cross sectional area of the jet nozzle
A_1	:	area of the outside cross section of the steel pipe
B_o	:	the width of the jet nozzle for two-dimensional case
C	:	the constant of jet diffusion in the infinite field
C_1	:	the value of C for the two-dimensional case
C_2	:	the value of C for three-dimensional case
D	:	the inside diameter of the main pipe
D_o	:	the inside diameter of the jet nozzle
D_1	:	the outside diameter of the jet conduit
E	:	the energy flux at any section
E_o	:	the energy flux at the efflux section
G	:	the acceleration of gravity of the earth, 32.2 ft/sec^2
H_n	:	the head reading of the n^{th}. point on the piezometer
HD	:	the water level reading at the downstream tank
HU	:	the water level reading at the head tank
HW	:	the head reading at the weir, HW=HD-2.251'
HWL	:	the water level difference between upstream and downstream tanks
H_1	:	the heading reading of the jet nozzle on the pitot-static tube
K_m	:	the coefficient of the m^{th} pitot tube, m= 1, 2, 3, and 4
L_o	:	a linear dimension characterizing the particular outlet form
M	:	the momentum flux at any section
M_o	:	the momentum flux at the efflux section
Q	:	the rate of total flow
Q_o	:	the rate of flow at the efflux section
Q_1	:	the rate of jet flow
Q_2	:	the rate of the secondary flow
R	:	the radial distance from the longitudinal central axis
T	:	the temperature of the water
U_{max}	:	the modified maximum velocity, $U_{max}=V_{max}-V_2$
U_o	:	the modified efflux velocity, $U_o= V_1-V_2$

U_{mn}	:	the modified velocity, $U_{mn} = V_{mn} - V_2$
V	:	mean velocity at any point in flow
V_m	:	the mean velocity in the pipe used in the calibration of the pitot tube
V_{max}	:	the maximum velocity at the central axis
V_{mn}	:	the velocity at the n^{th} point of the m^{th} section
V_x	:	mean velocity in X direction
V_o	:	the efflux velocity of the jet
V_y	:	mean velocity in Y direction
V_z	:	mean velocity in Z direction
V_p	:	the pitot-tube-measured mean velocity used in the calibration of the pitot tube
V_1	:	the velocity of the jet
V_2	:	the mean velocity of the secondary flow
X	:	the distrance along the central axis from the jet
Y	:	the coordinate distance in Y direction
X_m	:	the sections of the velocity measurements, m= 1, 2, 3, and 4
Z	:	the coordinate distance in Z direction
σ	:	the deviation of the Gaussian normal function

APPENDIX II

FLOW DIAGRAMS USED FOR DIGITAL COMPUTER PROGRAM

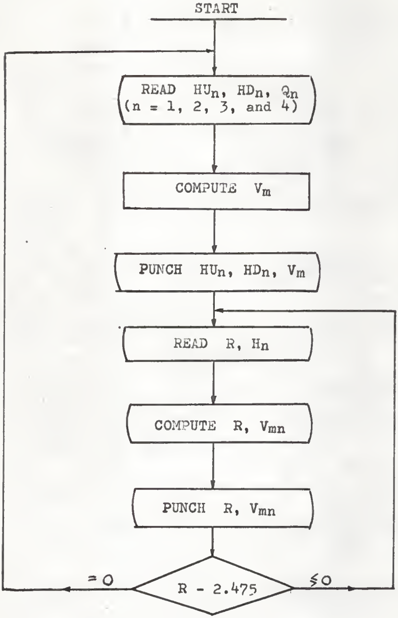


Fig. 25 The Determination of the Coefficients of pitot-static tubes

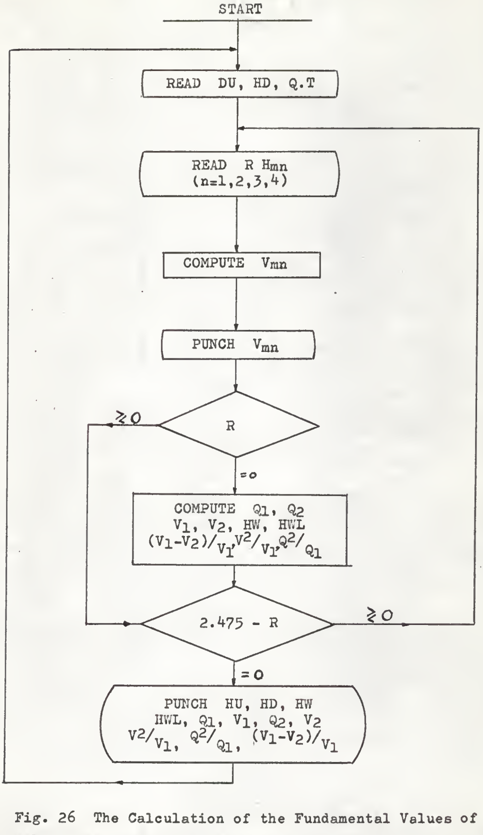


Fig. 26 The Calculation of the Fundamental Values of velocities, discharge, velocity ratio and discharge ratio.

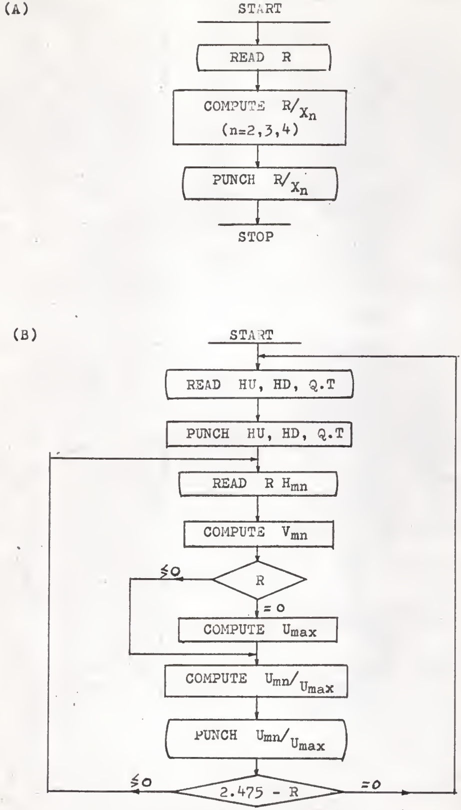
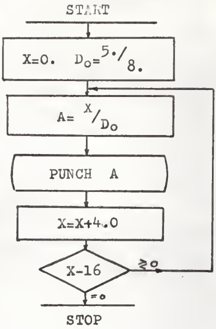


Fig. 27 The calculation of $R/_X$ and $U_{mn}/_{U_{max}}$



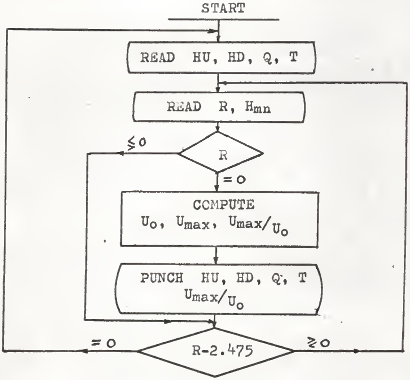


Fig. 28 The calculation of X/D_o and U_{max}/U_o

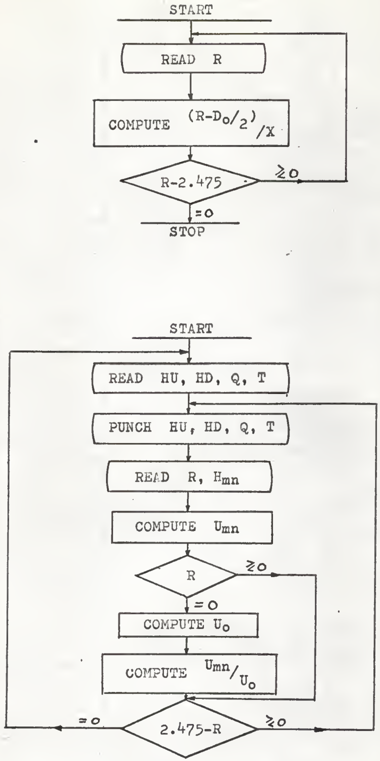


Fig. 29 The calculation of $(R-D_o/2)/_X$ and $U_{mn}/_{U_o}$

APPENDIX III

COMPUTER PROGRAMS USED FOR THIS STUDY

```
C  C  THE CALIBRATION OF THE COEFFICIENTS OF THE PITOT TUBES
  101 READ 1,HU1,HU2,HU3,HU4
      READ 2,HD1,HD2,HD3,HD4
      READ 3,Q1,Q2,Q3,Q4
      V1=Q1/(3.1416*(((5./12.)**2-(11./(16.*12.))**2)/4.))
      V2=Q2/(3.1416*((5./12.)**2/4.))
      V3=Q3/(3.1416*((5./12.)**2/4.))
      V4=Q4/(3.1416*((5./12.)**2/4.))
      PUNCH 4,HU1,HU2,HU3,HU4
      PUNCH 5,HD1,HD2,HD3,HD4
      PUNCH 6,Q1,Q2,Q3,Q4
      PUNCH 7,V1,V2,V3,V4
      PUNCH 9
  102 READ 8,R,H1,H2,H3,H4
      U1=SQRT(2.*32.2*H1/12.)
      U2=SQRT(2.*32.2*H2/12.)
      U3=SQRT(2.*32.2*H3/12.)
      U4=SQRT(2.*32.2*H4/12.)
      PUNCH 10,R,U1,U2,U3,U4
      IF (2.475-R) 102,103,102
  103 GO TO 101
    1 FORMAT (F8.3,F8.3,F8.3,F8.3)
    2 FORMAT (F8.3,F8.3,F8.3,F8.3)
    3 FORMAT (F8.3,F8.3,F8.3,F8.3)
    4 FORMAT (///5X4HHU1=F6.3,3X4HHU2=F6.3,3X4HHU3=F6.3,3X4HHU4=F6.3)
    5 FORMAT (5X4HHD1=F6.3,3X4HHD2=F6.3,3X4HHD3=F6.3,3X4HHD4=F6.3)
    6 FORMAT (6X3HQ1=F6.3,4X3HQ2=F6.3,4X3HQ3=F6.3,4X3HQ4=F6.3)
    7 FORMAT (6X3HV1=F6.3,4X3HV2=F6.3,4X3HV3=F6.3,4X3HV4=F6.3)
    8 FORMAT (F8.3,F8.3,F8.3,F8.3,F8.3)
    9 FORMAT (/53H       R          V1N         V2N         V3N         V4N)
   10 FORMAT (3XF8.3,3XF8.3,3XF8.3,3XF8.3,3XF8.3)
      END
```

```
C   C  THE STUDY OF THE CONFINED JET (1)
  101 READ 1,HU,HD,Q,T
      PUNCH 2
  102 READ 3,R,H1,H2,H3,H4
      U1=0.986*SQRT(2.*32.2*H1/12.)
      U2=0.956*SQRT(2.*32.2*H2/12.)
      U3=0.932*SQRT(2.*32.2*H3/12.)
      U4=0.961*SQRT(2.*32.2*H4/12.)
      PUNCH 4,R,U1,U2,U3,U4
  103 IF (R) 105,104,105
  104 Q1=U1*(3.1416*(5./8./12.)**2/4.)
      Q2=Q-Q1
      V2=Q2/(3.1416/4.*((5./12.)**2-(11./(16.*12.))**2))
      A=V2/U1
      AA=(U1-V2)/U1
      V1=U1
      B=Q2/Q1
      HW=HD-2.251
      HWL=(HU-7.91)-12.*HW
  105 IF (2.475-R) 102,106,102
  106 PUNCH 5,HU,HD,HW,HWL,T
      PUNCH 6,Q1,V1,Q2,V2,Q
      PUNCH 7,A,B,AA
      GO TO 101
    1 FORMAT (F8.3,F8.3,F8.3,F8.3)
    2 FORMAT (///51H    R           V1N          V2N         V3N         V4N)
    3 FORMAT (F8.3,F8.3,F8.3,F8.3,F8.3)
    4 FORMAT (3XF6.3,5XF6.3,5XF6.3,5XF6.3,5XF6.3)
    5 FORMAT (3HHU=F6.3,4X3HHD=F6.3,4X3HHW=F6.3,4X4HHWL=F6.3,5X2HT=F5.2)
    6 FORMAT (3HQ1=F6.3,4X3HV1=F6.3,4X3HQ2=F6.3,5X3HV2=F6.3,5X2HQ=F6.3)
    7 FORMAT (6HV2/V1=F8.3,4X6HQ2/Q1=F8.3,4X11H(V1-V2)/V1=F8.3)
      END
```

```
C  C THE STUDY OF THE CONFINED JET (2A)
  101 PUNCH 1
      READ 2,HU,HD,Q,T
      PUNCH 3
  102 READ 4,R,H1,H2,H3,H4
      RX2=R/4.
      RX3=R/8.
      RX4=R/12.
      PUNCH 5,R,RX2,RX3,RX4
  103 IF (R-2.475) 102,104,102
  104 STOP
    1 FORMAT (/27H      THE CALCULATION OF R/X)
    2 FORMAT (F8.3,F8.3,F8.3,F8.3)
    3 FORMAT (/49H    R        R/X1        R/X2        R/X3        R/X4)
    4 FORMAT (F8.3,F8.3,F8.3,F8.3,F8.3)
    5 FORMAT (3XF6.3,13XF6.3,5XF6.3,5XF6.3)
      END


C  C THE STUDY OF THE CONFINED JET (3A)
  101 PUNCH 1
      X=0.
      D0=5./8.
  102 A=X/D0
      PUNCH 2,A
      X=X+4.
      IF (X-16.) 102,103,102
  103 STOP
    1 FORMAT (/27H    THE CALCULATION OF X/D0)
    2 FORMAT (5XF8.3)
      END
```

```
C  C  THE STUDY OF THE CONFINED JET (4A)
  101 PUNCH 1
      PUNCH 2
  102 READ 3,R
      RXD2=(R-5./16.)/4.
      RXD3=(R-5./16.)/8.
      RXD4=(R-5./16.)/12.
      PUNCH 4,R,RXD2,RXD3,RXD4
  103 IF (R-2.475) 102,104,102
  104 STOP
    1 FORMAT (/33H    THE CALCULATION OF (R-DO/2)/X)
    2 FORMAT (/48H     R        X1            X2            X3            X4)
    3 FORMAT (F8.3)
    4 FORMAT (3XF6.3,13XF6.3,5XF6.3,5XF6.3)
      END
```

```
C   C THE STUDY OF THE CONFINED JET (2B)
      PUNCH 1
  101 READ 2,HU,HD,Q,T
      PUNCH 3,HU,HD,Q,T
      PUNCH 5
  102 READ 4,R,H1,H2,H3,H4
      U1=0.986*SQRT(2.*32.2*H1/12.)
      U2=0.956*SQRT(2.*32.2*H2/12.)
      U3=0.932*SQRT(2.*32.2*H3/12.)
      U4=0.961*SQRT(2.*32.2*H4/12.)
  103 IF (R) 105,104,105
  104 Q1=U1*(3.1416*(5./8./12.)**2/4.)
      Q2=Q-Q1
      V2=Q2/(3.1416/4.*((5./12.)**2-(11./(16.*12.))**2))
      UM1=U1-V2
      UM2=U2-V2
      UM3=U3-V2
      UM4=U4-V2
  105 UU1=(U1-V2)/UM1
      UU2=(U2-V2)/UM2
      UU3=(U3-V2)/UM3
      UU4=(U4-V2)/UM4
      PUNCH 6,R,UU1,UU2,UU3,UU4
      IF (2.475-R) 102,106,102
  106 GO TO 101
    1 FORMAT (/32H     THE CALCULATION OF UMN/MAX)
    2 FORMAT (F8.3,F8.3,F8.3,F8.3,F8.3)
    3 FORMAT (///3X3HHU=F6.3,4X3HHD=F6.3,4X2HQ=F6.3,4X2HT=F5.2)
    4 FORMAT (F8.3,F8.3,F8.3,F8.3,F8.3)
    5 FORMAT (/51H       R       U1N/UMAX  U2N/UMAX  U3N/UMAX  U4N/UMAX)
    6 FORMAT (3XF6.3,3XF6.3,4XF6.3,4XF6.3,4XF6.3)
      END
```

```
C   C THE STUDY CF THE CCNFINED JET (3B)
      PUNCH 1
  101 READ 2,HU,HD,Q,T
  102 READ 3,R,H1,H2,H3,H4
      IF (R) 105,110,105
  110 U1=0.986*SQRT(2.*32.2*H1/12.)
      U2=0.956*SQRT(2.*32.2*H2/12.)
      U3=0.932*SQRT(2.*32.2*H3/12.)
      U4=0.961*SQRT(2.*32.2*H4/12.)
  104 Q1=U1*(3.1416*(5./8./12.)**2/4.)
      Q2=Q-Q1
      V2=Q2/(3.1416/4.*((5./12.)**2-(11./(16.*12.))**2))
      UM1=U1-V2
      UM2=U2-V2
      UM3=U3-V2
      UM4=U4-V2
      UM01=UM1/UM1
      UM02=UM2/UM1
      UM03=UM3/UM1
      UM04=UM4/UM1
      PUNCH 4,HU,HD,Q,T
      PUNCH 5
      PUNCH 6, UM01,UM02,UM03,UM04
  105 IF (R-2.475) 102,106,102
  106 GC TC 101
    1 FCRMAT (/30H    THE CALCULATICN CF UMAX/U0)
    2 FCRMAT (F8.3,F8.3,F8.3,F8.3)
    3 FCRMAT (F8.3,F8.3,F8.3,F8.3,F8.3)
    4 FCRMAT (//3HHU=F6.3,4X3HHD=F6.3,4X2HQ=F6.3,4X2HT=F5.2)
    5 FCRMAT (53HVALUE CF UMAX/U0 AT          X1        X2        X3         X4)
    6 FCRMAT (25XF6.3,2XF6.3,2XF6.3,2XF6.3)
      END
```

```
C  C  THE STUDY OF THE CONFINED JET (4B)
  102 PUNCH 1
  103 READ 2,HU,HD,Q,T
      PUNCH 3,HU,HD,Q,T
      PUNCH 4
  105 READ 5,R,H1,H2,H3,H4
      U1=0.986*SQRT(2.*32.2*H1/12.)
      U2=0.956*SQRT(2.*32.2*H2/12.)
      U3=0.932*SQRT(2.*32.2*H3/12.)
      U4=0.961*SQRT(2.*32.2*H4/12.)
  110 IF (R) 115,111,115
  111 Q1=U1*(3.1416*(5./8./12.)**2/4.)
      Q2=Q-Q1
      V2=Q2/(3.1416/4.*((5./12.)**2-(11./(16.*12.))**2))
      V0=U1-V2
  115 UX1=U1-V2
      UX2=U2-V2
      UX3=U3-V2
      UX4=U4-V2
      VX01=UX1/V0
      VX02=UX2/V0
      VX03=UX3/V0
      VX04=UX4/V0
      PUNCH 6,R,VX01,VX02,VX03,VX04
      IF (2.475-R) 105,125,105
  125 GO TO 103
    1 FORMAT (31H      THE CALCULATION OF UMN/U0)
    2 FORMAT (F8.3,F8.3,F8.3,F8.3)
    3 FORMAT (///3X3HHU=F6.3,4X3HHD=F6.3,4X2HQ=F6.3,4X2HT=F5.2)
    4 FORMAT (/53H      R       U1N/U0      U2N/U0      U3N/U0      U4N/U0)
    5 FORMAT (F8.3,F8.3,F8.3,F8.3,F8.3)
    6 FORMAT (3XF6.3,5XF6.3,5XF6.3,5XF6.3,5XF6.3)
      END
```

APPENDIX IV

INPUT DATA TO DIGITAL COMPUTER PROGRAM

THE INPUT DATA FOR THE CALIBRATION OF PITOT TUBES

SERIES (1)

HU1	HU2	HU3	HU4
11.68	11.56	11.56	11.68

HD1	HD2	HD3	HD4
2.487	2.482	2.482	2.487

Q1	Q2	Q3	Q4
.230	0.223	0.223	0.230

R	H1N	H2N	H3N	H4N
.000		0.54	0.54	0.60
.500	0.63	0.60	0.55	0.63
1.000	0.62	0.59	0.54	0.62
1.500	0.61	0.58	0.54	0.62
2.000	0.57	0.53	0.50	0.59
2.475	0.50	0.32	0.34	0.35

SERIES (2)

HU1	HU2	HU3	HU4
16.58	16.85	16.85	16.58

HD1	HD2	HD3	HD4
2.643	2.651	2.651	2.643

Q1	Q2	Q3	Q4
.480	0.492	0.492	0.480

R	H1N	H2N	H3N	H4N
.000		2.40	2.59	2.68
.500	2.70	2.79	2.68	2.70
1.000	2.72	2.73	2.67	2.60
1.500	2.62	2.67	2.59	2.50
2.000	2.40	2.37	2.06	2.10
2.475	2.20	1.42	1.17	1.47

SERIES (3)

HU1	HU2	HU3	HU4
20.22	19.97	19.97	20.22

HD1	HD2	HD3	HD4
2.726	2.719	2.719	2.726

Q1	Q2	Q3	Q4
.621	0.615	0.615	0.621

R	H1N	H2N	H3N	H4N
.000		3.79	4.02	4.37
.500	4.50	4.39	4.26	4.37
1.000	4.46	4.35	4.22	4.28
1.500	4.30	4.24	4.13	4.17
2.000	3.93	3.97	3.71	3.50
2.475	3.44	2.10	1.72	2.33

THE INPUT DATA FOR THE MAIN STUDY

SERIES (1)

HU	HD	Q	T
7.500	2.299	0.033	23.0

R	H1N	H2N	H3N	H4N
.000	47.00	19.25	4.80	0.90
.166		15.05	4.25	
.333		5.75	3.90	
.500	0.08	1.75	2.80	0.76
1.000	0.03	0.04	0.60	0.29
1.500	0.02	0.03	0.00	0.04
2.000	0.02	0.02	0.10	0.09
2.475	0.00	0.04	0.00	0.02

HU	HD	Q	T
7.700	2.295	0.030	24.00

R	H1N	H2N	H3N	H4N
.000	37.37	15.70	3.40	0.67
.166		12.00	3.20	
.333		4.90	2.65	
.500	0.04	1.25	1.90	0.52
1.000	0.02	0.03	0.41	0.23
1.500	0.02	0.02	0.00	0.08
2.000	0.02	0.02	0.05	0.06
2.475	0.02	0.05	0.01	0.02

HU	HD	Q	T
7.820	2.289	0.024	25.1

R	H1N	H2N	H3N	H4N
.000	25.08	13.05	2.75	0.70
.166		7.30	2.45	
.333		2.05	2.20	
.500	0.08	0.72	1.50	0.40
1.000	0.06	0.02	0.25	0.15
1.500	0.04	0.00	0.03	0.02
2.000	0.06	0.02	0.03	0.04
2.475	0.04	0.05	0.00	0.00

HU	HD	Q	T
7.90	2.283	0.020	26.00

R	H1N	H2N	H3N	H4N
.000	17.45	8.75	2.40	0.50
.166		4.00		
.333		1.85		
.500	0.03	0.41	1.40	0.33
1.000	0.03	0.02	0.25	0.06
1.500	0.01	0.02	0.02	0.01
2.000	0.03	0.02	0.00	0.02
2.475	0.01	0.00	0.00	0.00

HU	HD	Q	T
8.000	2.275	0.014	26.5

R	H1N	H2N	H3N	H4N
.000	8.57	4.06	1.05	0.20
.166		3.05		
.333		1.05		
.500	0.01	0.40	0.70	0.21
1.000	0.00	0.04	0.32	0.07
1.500	0.00	0.01	0.05	0.02
2.000	0.00	0.01	0.02	0.01
2.475	0.00	0.00	0.00	0.01

SERIES (2)

HU	HD	Q	T
11.80	2.523	0.283	17.5

R	H1N	H2N	H3N	H4N
.000	40.35	20.90	9.65	4.55
.166		8.85	6.90	4.05
.333		2.64	3.90	2.49
.500	0.83	0.82	3.35	2.25
1.000	0.79	0.78	0.80	1.15
1.500	0.79	0.73	0.67	0.66
2.000	0.72	0.67	0.63	0.50
2.475	0.66	0.50	0.40	0.38

HU	HD	Q	T
11.96	2.519	0.277	17.5

R	H1N	H2N	H3N	H4N
.000	32.10	17.30	7.67	3.73
.166		6.08	6.45	3.30
.333		1.79	3.97	2.70
.500	0.87	0.81	2.39	1.90
1.000	0.82	0.74	0.75	0.93
1.500	0.79	0.71	0.69	0.68
2.000	0.74	0.67	0.65	0.55
2.475	0.67	0.48	0.41	0.34

HU	HD	Q	T
12.25	2.525	0.285	17.5

R	H1N	H2N	H3N	H4N
.000	23.55	11.92	6.70	3.40
.166		4.37	5.55	3.00
.333		1.42	4.09	2.60
.500	0.86	0.92	2.01	2.17
1.000	0.87	0.81	0.84	0.98
1.500	0.86	0.79	0.74	0.75
2.000	0.84	0.78	0.71	0.68
2.475	0.80	0.60	0.47	0.47

HU	HD	Q	T
13.00	2.547	0.319	17.5

R	H1N	H2N	H3N	H4N
.000	16.00	8.49	5.12	3.10
.166		3.90	4.20	2.83
.333		1.38	2.85	2.26
.500	1.08	1.18	2.14	1.92
1.000	1.12	1.06	1.10	1.23
1.500	1.08	1.05	1.04	1.05
2.000	1.06	1.03	1.04	1.01
2.475	1.03	0.78	0.65	0.63

HU	HD	Q	T
13.45	2.557	0.333	17.5

R	H1N	H2N	H3N	H4N
.000	8.66	4.86	3.39	2.15
.166		2.65	2.70	1.91
.333		1.67	1.90	1.77
.500	1.23	1.33	1.45	1.64
1.000	1.24	1.22	1.24	1.35
1.500	1.24	1.20	1.22	1.29
2.000	1.21	1.18	1.22	1.20
2.475	1.13	0.93	0.80	0.81

SERIES (3)

HU	HD	Q	T
14.87	2.620	0.438	19.8

R	H1N	H2N	H3N	H4N
.000	50.73	30.42	13.43	7.58
.166		11.06	10.48	6.19
.333		3.15	6.23	5.06
.500	2.01	2.08	4.95	3.51
1.000	1.96	2.01	1.79	1.89
1.500	1.96	1.98	1.79	1.70
2.000	1.89	1.89	1.74	1.63
2.475	1.68	1.25	1.17	1.07

HU	HD	Q	T
14.62	2.611	0.422	20.0

R	H1N	H2N	H3N	H4N
.000	42.43	25.30	11.39	6.55
.166		11.15	9.05	5.68
.333		2.62	5.03	4.56
.500	1.91	1.81	3.19	3.43
1.000	1.85	1.75	1.68	1.81
1.500	1.83	1.74	1.66	1.63
2.000	1.78	1.72	1.61	1.53
2.475	1.70	1.33	1.03	0.87

HU	HD	Q	T
14.79	2.610	0.420	20.4

R	H1N	H2N	H3N	H4N
.000	33.60	20.00	9.57	5.46
.166		8.65	7.50	4.75
.333		2.72	4.59	3.61
.500	1.94	1.84	2.85	3.08
1.000	1.89	1.82	1.71	1.85
1.500	1.88	1.80	1.71	1.75
2.000	1.84	1.74	1.65	1.57
2.475	1.66	1.42	1.02	1.01

HU	HD	Q	T
14.86	2.605	0.415	20.4

R	H1N	H2N	H3N	H4N
.000	25.40	14.65	7.80	4.53
.166		6.35	6.32	4.14
.333		2.40	4.05	3.38
.500	1.91	1.81	2.62	2.72
1.000	1.88	1.78	1.75	1.83
1.500	1.81	1.77	1.72	1.73
2.000	1.77	1.72	1.62	1.69
2.475	1.63	1.26	1.04	1.17

HU	HD	Q	T
14.98	2.607	0.416	21.0

R	H1N	H2N	H3N	H4N
.000	15.59	9.64	5.53	3.25
.166		4.12	3.83	3.00
.333		2.03	3.12	2.56
.500	1.92	1.86	2.07	2.21
1.000	1.88	1.81	1.76	1.86
1.500	1.85	1.80	1.75	1.81
2.000	1.79	1.73	1.72	1.69
2.475	1.64	1.32	1.04	1.07

HU	HD	Q	T
15.02	2.604	0.412	21.0

R	H1N	H2N	H3N	H4N
.000	7.25	5.05	3.12	2.44
.166		2.58	2.75	2.24
.333		1.89	2.13	2.11
.500	1.93	1.86	1.95	1.95
1.000	1.91	1.85	1.84	1.89
1.500	1.84	1.83	1.81	1.89
2.000	1.82	1.76	1.79	1.83
2.475	1.66	1.32	1.13	1.59

SERIES (4)

HU	HD	Q	T
22.60	2.777	0.728	21.0

R	H1N	H2N	H3N	H4N
.000	40.48	27.12	14.60	9.75
.166		21.95	14.02	9.62
.333		9.35	11.25	8.90
.500	5.31	6.05	8.25	7.50
1.000	5.66	5.78	5.84	6.10
1.500	5.66	5.61	5.56	5.50
2.000	5.30	5.35	4.95	4.32
2.475	4.65	3.12	2.42	2.39

HU	HD	Q	T
22.69	2.775	0.725	21.0

R	H1N	H2N	H3N	H4N
.000	31.16	20.10	12.00	8.75
.166		12.30	11.00	8.30
.333		6.74	9.45	7.70
.500	5.75	5.98	7.22	7.08
1.000	5.63	5.72	5.85	6.13
1.500	5.54	5.70	5.47	6.04
2.000	5.29	5.26	4.98	4.57
2.475	4.73	3.37	2.62	2.50

Q1	Q2	Q3	Q4
22.68	2.772	0.721	21.0

R	H1N	H2N	H3N	H4N
.000	20.09	13.54	9.15	7.10
.166		8.24	8.76	7.12
.333		6.80	7.15	6.78
.500	5.38	5.76	6.49	6.57
1.000	5.57	5.79	5.71	5.85
1.500	5.65	5.74	5.57	5.71
2.000	5.19	5.17	5.00	4.98
2.475	4.48	3.78	2.53	2.37

Q1	Q2	Q3	Q4
22.70	2.770	0.715	21.0

R	H1N	H2N	H3N	H4N
.000	11.30	8.40	6.58	6.08
.166		6.12	6.35	6.03
.333		5.42	5.95	5.97
.500	5.43	5.62	5.68	5.91
1.000	5.59	5.73	5.66	5.79
1.500	5.62	5.70	5.56	5.61
2.000	5.33	5.23	5.13	4.05
2.475	4.33	2.98	2.43	2.28

APPENDIX V

OUTPUT DATA FROM DIGITAL COMPUTER PROGRAM

C C THE CALIBRATION OF THE COEFFICIENTS OF THE PITOT TUBES

HU1=11.680	HU2=11.560	HU3=11.560	HU4=11.680
HD1= 2.487	HD2= 2.482	HD3= 2.482	HD4= 2.487
Q1= .230	Q2= .223	Q3= .223	Q4= .230
V1= 1.719	V2= 1.635	V3= 1.635	V4= 1.687

R	VX1	VX2	VX3	VX4
.000	0.000	1.702	1.702	1.794
.500	1.839	1.794	1.718	1.839
1.000	1.824	1.779	1.702	1.824
1.500	1.809	1.764	1.702	1.824
2.000	1.749	1.687	1.638	1.779
2.475	1.638	1.310	1.351	1.371

VP1= 1.690	VP2= 1.560	VP3= 1.530	VP4= 1.610
K1= .979	K2= .955	K3= .936	K4= .954

HU1=16.580	HU2=16.850	HU3=16.850	HU4=16.580
HD1= 2.643	HD2= 2.651	HD3= 2.651	HD4= 2.643
Q1= .480	Q2= .492	Q3= .492	Q4= .480
V1= 3.588	V2= 3.608	V3= 3.608	V4= 3.520

R	VX1	VX2	VX3	VX4
.000	0.000	3.589	3.728	3.792
.500	3.807	3.869	3.792	3.807
1.000	3.821	3.828	3.785	3.735
1.500	3.750	3.785	3.728	3.663
2.000	3.589	3.566	3.325	3.357
2.475	3.436	2.761	2.506	2.809

VP1= 3.550	VP2= 3.470	VP3= 3.350	VP4= 3.380
K1= .989	K2= .961	K3= .929	K4= .961

HU1=20.220	HU2=19.970	HU3=19.970	HU4=20.220
HD1= 2.726	HD2= 2.719	HD3= 2.719	HD4= 2.726
Q1= .621	Q2= .615	Q3= .615	Q4= .621
V1= 4.642	V2= 4.510	V3= 4.510	V4= 4.554

R	VX1	VX2	VX3	VX4
.000	0.000	4.510	4.645	4.843
.500	4.914	4.854	4.781	4.843
1.000	4.892	4.832	4.759	4.793
1.500	4.804	4.770	4.708	4.731
2.000	4.592	4.616	4.462	4.334
2.475	4.297	3.357	3.038	3.536

VP1= 4.600	VP2= 4.300	VP3= 4.200	VP4= 4.400
K1= .990	K2= .953	K3= .931	K4= .969

MEAN OF THE COEFFICIENTS

K1= .986	K2= .956	K3= .932	K4= .961

C C THE STUDY OF THE CONFINED JET (1)

THE CALCULATION OF THE FUNDAMENTAL VALUES (SERIES 1)

R	V1N	V2N	V3N	V4N
.000	15.660	9.717	4.730	2.112
.166		8.592	4.451	
.333		5.311	4.264	
.500	.646	2.930	3.613	1.941
1.000	.396	.443	1.672	1.199
1.500	.323	.384	0.000	.445
2.000	.323	.313	.683	.668
2.475	0.000	.443	0.000	.315

HU= 7.500 HD= 2.299 HW= .048 HWL= -.986 T=23.00
Q1= .033 V1=15.660 Q2= 0.000 V2= 0.000 Q= .033
V2/V1= 0.000 Q2/Q1= 0.000 (V1-V2)/V1= 1.000

R	V1N	V2N	V3N	V4N
.000	13.963	8.775	3.981	1.822
.166		7.672	3.862	
.333		4.902	3.515	
.500	.457	2.476	2.976	1.605
1.000	.323	.384	1.382	1.068
1.500	.323	.313	0.000	.630
2.000	.323	.313	.483	.545
2.475	.323	.495	.216	.315

HU= 7.700 HD= 2.295 HW= .044 HWL= -.738 T=24.00
Q1= .030 V1=13.963 Q2= 0.000 V2= 0.000 Q= .030
V2/V1= 0.000 Q2/Q1= 0.000 (V1-V2)/V1= 1.000

R	V1N	V2N	V3N	V4N
.000	11.439	8.000	3.580	1.863
.166		5.984	3.379	
.333		3.171	3.202	
.500	.646	1.879	2.644	1.408
1.000	.560	.313	1.080	.862
1.500	.457	0.000	.374	.315
2.000	.560	.313	.374	.445
2.475	.457	.495	0.000	0.000

HU= 7.820 HD= 2.289 HW= .038 HWL= -.546 T=25.10
Q1= .024 V1=11.439 Q2= 0.000 V2= 0.000 Q= .024
V2/V1= 0.000 Q2/Q1= 0.000 (V1-V2)/V1= 1.000

R	V1N	V2N	V3N	V4N
.000	9.542	6.551	3.345	1.574
.166		4.429		
.333		3.012		
.500	.396	1.418	2.555	1.279
1.000	.396	.313	1.080	.545
1.500	.228	.313	.305	.223
2.000	.396	.313	0.000	.315
2.475	.228	0.000	0.000	0.000

HU= 7.900 HD= 2.283 HW= .032 HWL= -.394 T=26.00
Q1= .020 V1= 9.542 Q2= 0.000 V2= 0.000 Q= .020
V2/V1= 0.000 Q2/Q1= 0.000 (V1-V2)/V1= 1.000

R	V1N	V2N	V3N	V4N
.000	6.687	4.462	2.212	.996
.166		3.868		
.333		2.269		
.500	.228	1.401	1.806	1.020
1.000	0.000	.443	1.221	.589
1.500	0.000	.221	.483	.315
2.000	0.000	.221	.305	.223
2.475	0.000	0.000	0.000	.223

HU= 8.000 HD= 2.275 HW= .024 HWL= -.198 T=26.50
Q1= .014 V1= 6.687 Q2= 0.000 V2= 0.000 Q= .014
V2/V1= 0.000 Q2/Q1= 0.000 (V1-V2)/V1= 1.000

THE CALCULATION OF THE FUNDAMENTAL VALUES (SERIES 2)

R	V1N	V2N	V3N	V4N
.000	14.509	10.125	6.707	4.749
.166		6.588	5.671	4.480
.333		3.598	4.264	3.513
.500	2.081	2.005	3.952	3.339
1.000	2.030	1.956	1.931	2.387
1.500	2.030	1.892	1.767	1.809
2.000	1.938	1.813	1.714	1.574
2.475	1.856	1.566	1.366	1.372

HU=11.800 HD= 2.523 HW= .272 HWL= .626 T=17.50
Q1= .031 V1=14.509 Q2= .252 V2= 1.884 Q= .283
V2/V1= .130 Q2/Q1= 8.155 (V1-V2)/V1= .870

R	V1N	V2N	V3N	V4N
.000	12.941	9.212	5.980	4.300
.166		5.461	5.483	4.044
.333		2.963	4.302	3.658
.500	2.131	1.993	3.338	3.069
1.000	2.068	1.905	1.870	2.147
1.500	2.030	1.866	1.793	1.836
2.000	1.965	1.813	1.741	1.651
2.475	1.870	1.534	1.382	1.298

HU=11.960 HD= 2.519 HW= .268 HWL= .834 T=17.50
Q1= .028 V1=12.941 Q2= .249 V2= 1.865 Q= .277
V2/V1= .144 Q2/Q1= 9.046 (V1-V2)/V1= .856

R	V1N	V2N	V3N	V4N
.000	11.085	7.646	5.589	4.105
.166		4.630	5.086	3.856
.333		2.639	4.366	3.590
.500	2.118	2.124	3.061	3.279
1.000	2.131	1.993	1.979	2.204
1.500	2.118	1.968	1.857	1.928
2.000	2.093	1.956	1.819	1.836
2.475	2.043	1.715	1.480	1.526

HU=12.250 HD= 2.525 HW= .274 HWL= 1.052 T=17.50
Q1= .024 V1=11.085 Q2= .261 V2= 1.954 Q= .285
V2/V1= .176 Q2/Q1= 11.068 (V1-V2)/V1= .824

R	V1N	V2N	V3N	V4N
.000	9.137	6.453	4.885	3.920
.166		4.374	4.425	3.745
.333		2.602	3.645	3.347
.500	2.374	2.406	3.158	3.085
1.000	2.417	2.280	2.264	2.469
1.500	2.374	2.269	2.202	2.281
2.000	2.352	2.248	2.202	2.237
2.475	2.318	1.956	1.741	1.767

HU=13.000 HD= 2.547 HW= .296 HWL= 1.538 T=17.50
Q1= .019 V1= 9.137 Q2= .300 V2= 2.239 Q= .319
V2/V1= .245 Q2/Q1= 15.388 (V1-V2)/V1= .755

R	V1N	V2N	V3N	V4N
.000	6.722	4.882	3.975	3.264
.166		3.605	3.548	3.077
.333		2.862	2.976	2.962
.500	2.533	2.554	2.600	2.851
1.000	2.544	2.446	2.404	2.587
1.500	2.544	2.426	2.385	2.529
2.000	2.513	2.406	2.385	2.439
2.475	2.428	2.136	1.931	2.004

HU=13.450 HD= 2.557 HW= .306 HWL= 1.868 T=17.50
Q1= .014 V1= 6.722 Q2= .319 V2= 2.382 Q= .333
V2/V1= .354 Q2/Q1= 22.252 (V1-V2)/V1= .646

THE CALCULATION OF THE FUNDAMENTAL VALUES (SERIES 3)

R	V1N	V2N	V3N	V4N
.000	16.269	12.215	7.912	6.129
.166		7.365	6.990	5.539
.333		3.931	5.389	5.008
.500	3.238	3.194	4.804	4.171
1.000	3.198	3.140	2.889	3.061
1.500	3.198	3.116	2.889	2.903
2.000	3.140	3.045	2.848	2.842
2.475	2.961	2.476	2.335	2.303

HU=14.870 HD= 2.620 HW= .369 HWL= 2.532 T=19.80
Q1= .035 V1=16.269 Q2= .403 V2= 3.015 Q= .438
V2/V1= .185 Q2/Q1= 11.636 (V1-V2)/V1= .815

R	V1N	V2N	V3N	V4N
.000	14.879	11.140	7.287	5.698
.166		7.395	6.495	5.306
.333		3.585	4.842	4.754
.500	3.157	2.980	3.856	4.123
1.000	3.107	2.930	2.798	2.995
1.500	3.090	2.921	2.782	2.842
2.000	3.047	2.905	2.740	2.754
2.475	2.978	2.554	2.191	2.077

HU=14.620 HD= 2.611 HW= .360 HWL= 2.390 T=20.00
Q1= .032 V1=14.879 Q2= .390 V2= 2.918 Q= .422
V2/V1= .196 Q2/Q1= 12.312 (V1-V2)/V1= .804

R	V1N	V2N	V3N	V4N
.000	13.240	9.904	6.679	5.202
.166		6.514	5.913	4.852
.333		3.653	4.626	4.230
.500	3.181	3.004	3.645	3.907
1.000	3.140	2.988	2.823	3.028
1.500	3.132	2.971	2.823	2.945
2.000	3.098	2.921	2.773	2.789
2.475	2.943	2.639	2.181	2.237

HU=14.790 HD= 2.610 HW= .359 HWL= 2.572 T=20.40
Q1= .028 V1=13.240 Q2= .392 V2= 2.929 Q= .420
V2/V1= .221 Q2/Q1= 13.889 (V1-V2)/V1= .779

R	V1N	V2N	V3N	V4N
.000	11.512	8.477	6.030	4.738
.166		5.581	5.428	4.530
.333		3.431	4.345	4.093
.500	3.157	2.980	3.495	3.672
1.000	3.132	2.955	2.856	3.012
1.500	3.073	2.946	2.832	2.928
2.000	3.039	2.905	2.748	2.894
2.475	2.916	2.486	2.202	2.408

HU=14.860 HD= 2.605 HW= .354 HWL= 2.702 T=20.40
Q1= .025 V1=11.512 Q2= .390 V2= 2.919 Q= .415
V2/V1= .254 Q2/Q1= 15.921 (V1-V2)/V1= .746

R	V1N	V2N	V3N	V4N
.000	9.019	6.876	5.077	4.013
.166		4.495	4.225	3.856
.333		3.155	3.814	3.562
.500	3.165	3.020	3.106	3.310
1.000	3.132	2.980	2.864	3.036
1.500	3.107	2.971	2.856	2.995
2.000	3.056	2.913	2.832	2.894
2.475	2.925	2.544	2.202	2.303

HU=14.980 HD= 2.607 HW= .356 HWL= 2.798 T=21.00
Q1= .019 V1= 9.019 Q2= .397 V2= 2.966 Q= .416
V2/V1= .329 Q2/Q1= 20.650 (V1-V2)/V1= .671

R	V1N	V2N	V3N	V4N
.000	6.150	4.977	3.814	3.478
.166		3.557	3.580	3.332
.333		3.045	3.151	3.234
.500	3.173	3.020	3.015	3.109
1.000	3.157	3.012	2.929	3.061
1.500	3.098	2.996	2.905	3.061
2.000	3.082	2.938	2.889	3.012
2.475	2.943	2.544	2.295	2.807

HU=15.020 HD= 2.604 HW= .353 HWL= 2.874 T=21.00
Q1= .013 V1= 6.150 Q2= .399 V2= 2.982 Q= .412
V2/V1= .485 Q2/Q1= 30.442 (V1-V2)/V1= .515

THE CALCULATION OF THE FUNDAMENTAL VALUES (SERIES 4)

R	V1N	V2N	V3N	V4N
.000	14.533	11.533	8.250	6.951
.166		10.376	8.084	6.905
.333		6.772	7.242	6.642
.500	5.264	5.447	6.201	6.097
1.000	5.434	5.324	5.218	5.498
1.500	5.434	5.246	5.091	5.221
2.000	5.259	5.123	4.804	4.627
2.475	4.926	3.912	3.359	3.442

HU=22.600 HD= 2.777 HW= .526 HWL= 8.378 T=21.00
Q1= .031 V1=14.533 Q2= .697 V2= 5.210 Q= .728
V2/V1= .359 Q2/Q1= 22.512 (V1-V2)/V1= .641

R	V1N	V2N	V3N	V4N
.000	12.751	9.929	7.479	6.585
.166		7.767	7.161	6.414
.333		5.750	6.637	6.178
.500	5.477	5.416	5.801	5.924
1.000	5.420	5.297	5.222	5.512
1.500	5.376	5.287	5.040	0.000
1.500	5.376	5.287	5.050	5.471
2.000	5.254	5.079	4.818	4.759
2.475	4.968	4.066	3.495	3.520

HU=22.690 HD= 2.775 HW= .524 HWL= 8.492 T=21.00
Q1= .027 V1=12.751 Q2= .698 V2= 5.216 Q= .725
V2/V1= .409 Q2/Q1= 25.688 (V1-V2)/V1= .591

R	V1N	V2N	V3N	V4N
.000	10.238	8.149	6.531	5.932
.166		6.357	6.390	5.940
.333		5.775	5.773	5.797
.500	5.298	5.315	5.500	5.706
1.000	5.391	5.329	5.159	5.385
1.500	5.429	5.306	5.096	5.320
2.000	5.204	5.036	4.828	4.968
2.475	4.835	4.306	3.434	3.427

HU=22.680 HD= 2.772 HW= .521 HWL= 8.518 T=21.00
Q1= .022 V1=10.238 Q2= .699 V2= 5.227 Q= .721
V2/V1= .510 Q2/Q1= 32.054 (V1-V2)/V1= .490

R	V1N	V2N	V3N	V4N
.000	7.678	6.419	5.538	5.489
.166		5.479	5.441	5.467
.333		5.156	5.267	5.440
.500	5.323	5.250	5.146	5.412
1.000	5.401	5.301	5.137	5.357
1.500	5.415	5.287	5.091	5.273
2.000	5.273	5.065	4.890	4.480
2.475	4.753	3.823	3.366	3.362

HU=22.700 HD= 2.770 HW= .519 HWL= 8.562 T=21.00
Q1= .016 V1= 7.678 Q2= .699 V2= 5.222 Q= .715
V2/V1= .680 Q2/Q1= 42.707 (V1-V2)/V1= .320

C C THE STUDY OF THE CONFINED JET (2A)

THE CALCULATION OF R/X

R	R/X1	R/X2	R/X3	R/X4
.000		0.000	0.000	0.000
.166		.042	.021	.014
.333		.083	.042	.028
.500		.125	.063	.042
1.000		.250	.125	.083
1.500		.375	.188	.125
2.000		.500	.250	.167
2.475		.619	.309	.206

C C THE STUDY OF THE CONFINED JET (3A)

THE CALCULATION OF X/DO

VALUES AT X2=6.400 X3=12.800 X4=19.200

C C THE STUDY OF THE CONFINED JET (4A)

THE CALCULATION OF (R-DO/2)/X

R	X1	X2	X3	X4
.000		-.078	-.039	-.026
.166		-.037	-.018	-.012
.333		.005	.003	.002
.500		.047	.023	.016
1.000		.172	.086	.057
1.500		.297	.148	.099
2.000		.422	.211	.141
2.475		.541	.270	.180

C C THE STUDY OF THE CONFINED JET (2B)

THE CALCULATION OF UMN/UMAX (SERIES 1)

HU= 7.500 HD= 2.299 Q= .033 T=23.00

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.884	.941	
.333		.547	.901	
.500	.041	.302	.764	.919
1.000	.025	.046	.354	.568
1.500	.021	.040	.000	.212
2.000	.021	.033	.145	.317
2.475	.000	.046	.000	.150

HU= 7.700 HD= 2.295 Q= .030 T=24.00

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.874	.970	
.333		.559	.883	
.500	.033	.282	.747	.881
1.000	.023	.044	.347	.585
1.500	.023	.035	-.000	.345
2.000	.023	.035	.121	.299
2.475	.023	.056	.054	.172

HU= 7.820 HD= 2.289 Q= .024 T=25.10

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.748	.944	
.333		.397	.895	
.500	.057	.235	.739	.756
1.000	.049	.039	.302	.464
1.500	.040	.000	.105	.170
2.000	.049	.039	.105	.240
2.475	.040	.062	.000	.001

HU= 7.900 HD= 2.283 Q= .020 T=26.00

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.676		
.333		.460		
.500	.042	.217	.764	.813
1.000	.042	.048	.323	.347
1.500	.024	.048	.092	.143
2.000	.042	.048	.000	.201
2.475	.024	.000	.000	.002

HU= 8.000 HD= 2.275 Q= .014 T=26.50

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.867		
.333		.509		
.500	.034	.314	.817	1.025
1.000	.000	.100	.552	.592
1.500	.000	.050	.219	.317
2.000	.000	.050	.139	.225
2.475	.000	.000	.000	.225

THE CALCULATION OF UMN/UMAX (SERIES 2)

HU=11.800 HD= 2.523 Q= .283 T=17.50

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.571	.785	.906
.333		.208	.493	.569
.500	.016	.015	.429	.508
1.000	.012	.009	.010	.176
1.500	.012	.000	-.024	-.026
2.000	.004	-.009	-.035	-.108
2.475	-.002	-.039	-.108	-.179

HU=11.960 HD= 2.519 Q= .277 T=17.50

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.489	.879	.895
.333		.150	.592	.737
.500	.024	.018	.358	.495
1.000	.018	.006	.001	.116
1.500	.015	.000	-.017	-.012
2.000	.009	-.007	-.030	-.088
2.475	.000	-.045	-.117	-.233

HU=12.250 HD= 2.525 Q= .285 T=17.50

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.470	.862	.884
.333		.120	.664	.760
.500	.018	.030	.305	.616
1.000	.019	.007	.007	.116
1.500	.018	.003	-.027	-.012
2.000	.015	.000	-.037	-.055
2.475	.010	-.042	-.130	-.199

HU=13.000 HD= 2.547 Q= .319 T=17.50

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.507	.826	.896
.333		.086	.531	.659
.500	.020	.040	.347	.503
1.000	.026	.010	.010	.137
1.500	.020	.007	-.014	.025
2.000	.016	.002	-.014	-.001
2.475	.011	-.067	-.188	-.281

HU=13.450 HD= 2.557 Q= .333 T=17.50

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.489	.732	.787
.333		.192	.373	.657
.500	.035	.069	.137	.531
1.000	.037	.026	.014	.232
1.500	.037	.018	.002	.166
2.000	.030	.009	.002	.064
2.475	.011	-.099	-.283	-.429

THE CALCULATION OF UMN/UMAX (SERIES 3)

HU=14.870 HD= 2.620 Q= .438 T=19.80

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.473	.812	.810
.333		.100	.485	.640
.500	.017	.019	.365	.371
1.000	.014	.014	-.026	.015
1.500	.014	.011	-.026	-.036
2.000	.009	.003	-.034	-.055
2.475	-.004	-.059	-.139	-.229

HU=14.620 HD= 2.611 Q= .422 T=20.00

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.545	.819	.859
.333		.081	.441	.661
.500	.020	.008	.215	.434
1.000	.016	.001	-.027	.028
1.500	.014	.000	-.031	-.027
2.000	.011	-.002	-.041	-.059
2.475	.005	-.044	-.166	-.303

HU=14.790 HD= 2.610 Q= .420 T=20.40

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.514	.796	.846
.333		.104	.452	.572
.500	.025	.011	.191	.430
1.000	.021	.008	-.028	.044
1.500	.020	.006	-.028	.007
2.000	.016	-.001	-.041	-.061
2.475	.001	-.042	-.199	-.304

HU=14.860 HD= 2.605 Q= .415 T=20.40

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.479	.806	.885
.333		.092	.458	.645
.500	.028	.011	.185	.414
1.000	.025	.006	-.020	.051
1.500	.018	.005	-.028	.005
2.000	.014	-.003	-.055	-.014
2.475	-.000	-.078	-.230	-.281

HU=14.980 HD= 2.607 Q= .416 T=21.00

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.391	.597	.850
.333		.048	.401	.569
.500	.033	.014	.066	.328
1.000	.027	.003	-.048	.067
1.500	.023	.001	-.052	.028
2.000	.015	-.014	-.064	-.069
2.475	-.007	-.108	-.362	-.633

HU=15.020 HD= 2.604 Q= .412 T=21.00

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.288	.720	.706
.333		.032	.203	.508
.500	.060	.019	.040	.256
1.000	.055	.015	-.064	.159
1.500	.037	.007	-.093	.159
2.000	.031	-.022	-.112	.060
2.475	-.012	-.219	-.825	-.352

THE CALCULATION OF UMN/UMAX (SERIES 4)

HU=22.600 HD= 2.777 Q= .728 T=21.00

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.817	.946	.973
.333		.247	.668	.822
.500	.006	.037	.326	.509
1.000	.024	.018	.002	.165
1.500	.024	.006	-.039	.006
2.000	.005	-.014	-.134	-.335
2.475	-.031	-.205	-.609	-1.016

HU=22.690 HD= 2.775 Q= .725 T=21.00

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.541	.859	.875
.333		.113	.628	.702
.500	.035	.042	.259	.517
1.000	.027	.017	.003	.216
1.500	.021	.015	-.078	-3.811
1.500	.021	.015	-.074	.186
2.000	.005	-.029	-.176	-.334
2.475	-.033	-.244	-.761	-1.239

HU=22.680 HD= 2.772 Q= .721 T=21.00

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.387	.892	1.012
.333		.188	.419	.808
.500	.014	.030	.210	.680
1.000	.033	.035	-.052	.224
1.500	.040	.027	-.100	.132
2.000	-.005	-.065	-.306	-.366
2.475	-.078	-.315	-1.374	-2.550

HU=22.700 HD= 2.770 Q= .715 T=21.00

R	U1N/UMAX	U2N/UMAX	U3N/UMAX	U4N/UMAX
.000	1.000	1.000	1.000	1.000
.166		.214	.691	.915
.333		-.056	.140	.813
.500	.041	.023	-.243	.711
1.000	.073	.066	-.272	.504
1.500	.078	.054	-.416	.189
2.000	.021	-.132	-1.052	-2.780
2.475	-.191	-1.170	-5.878	-6.970

C C THE STUDY OF THE CONFINED JET (3B)

THE CALCULATION OF UMAX/UO (SERIES 1)

```
HU= 7.500     HD= 2.299     Q=  .033      T=23.00
VALUE OF UMAX/UO AT           X1         X2        X3        X4
                             1.000       .621      .302      .135

HU= 7.700     HD= 2.295     Q=  .030      T=24.00
VALUE OF UMAX/UO AT           X1         X2        X3        X4
                             1.000       .628      .285      .130

HU= 7.820     HD= 2.289     Q=  .024      T=25.10
VALUE OF UMAX/UO AT           X1         X2        X3        X4
                             1.000       .699      .313      .163

HU= 7.900     HD= 2.283     Q=  .020      T=26.00
VALUE OF UMAX/UO AT           X1         X2        X3        X4
                             1.000       .687      .351      .165

HU= 8.000     HD= 2.275     Q=  .014      T=26.50
VALUE OF UMAX/UO AT           X1         X2        X3        X4
                             1.000       .667      .331      .149
```

THE CALCULATION OF UMAX/UO (SERIES 2)

```
HU=11.800     HD= 2.523     Q=  .283      T=17.50
VALUE OF UMAX/UO AT           X1         X2        X3        X4
                             1.000       .653      .382      .227

HU=11.960     HD= 2.519     Q=  .277      T=17.50
VALUE OF UMAX/UO AT           X1         X2        X3        X4
                             1.000       .663      .371      .220

HU=12.250     HD= 2.525     Q=  .285      T=17.50
VALUE OF UMAX/UO AT           X1         X2        X3        X4
                             1.000       .623      .398      .236
```

```
HU=13.000    HD= 2.547    Q=  .319    T=17.50
VALUE OF UMAX/UO AT        X1       X2       X3       X4
                          1.000    .611     .384     .244

HU=13.450    HD= 2.557    Q=  .333    T=17.50
VALUE OF UMAX/UO AT        X1       X2       X3       X4
                          1.000    .576     .367     .203
```

THE CALCULATION OF UMAX/UO (SERIES 3)

```
HU=14.870    HD= 2.620    Q=  .438    T=19.80
VALUE OF UMAX/UO AT        X1       X2       X3       X4
                          1.000    .694     .369     .235

HU=14.620    HD= 2.611    Q=  .422    T=20.00
VALUE OF UMAX/UO AT        X1       X2       X3       X4
                          1.000    .687     .365     .232

HU=14.790    HD= 2.610    Q=  .420    T=20.40
VALUE OF UMAX/UO AT        X1       X2       X3       X4
                          1.000    .676     .364     .220

HU=14.860    HD= 2.605    Q=  .415    T=20.40
VALUE OF UMAX/UO AT        X1       X2       X3       X4
                          1.000    .647     .362     .212

HU=14.980    HD= 2.607    Q=  .416    T=21.00
VALUE OF UMAX/UO AT        X1       X2       X3       X4
                          1.000    .646     .349     .173

HU=15.020    HD= 2.604    Q=  .412    T=21.00
VALUE OF UMAX/UO AT        X1       X2       X3       X4
                          1.000    .630     .263     .156
```

THE CALCULATION OF UMAX/UO (SERIES 4)

HU=22.600 HD= 2.777 Q= .728 T=21.00

VALUE OF UMAX/UO AT	X1	X2	X3	X4
	1.000	.678	.326	.187

HU=22.690 HD= 2.775 Q= .725 T=21.00

VALUE OF UMAX/UO AT	X1	X2	X3	X4
	1.000	.626	.300	.182

HU=22.680 HD= 2.772 Q= .721 T=21.00

VALUE OF UMAX/UO AT	X1	X2	X3	X4
	1.000	.583	.260	.141

HU=22.700 HD= 2.770 Q= .715 T=21.00

VALUE OF UMAX/UO AT	X1	X2	X3	X4
	1.000	.487	.129	.109

C C THE STUDY OF THE CONFINED JET (4B)

THE CALCULATION OF UMN/UO (SERIES 1)

HU= 7.500 HD= 2.299 Q= .033 T=23.00

R	U1N/UO	U2N/UO	U3N/UO	U4N/UO
.000	1.000	.621	.302	.135
.166		.549	.284	
.333		.339	.272	
.500	.041	.187	.231	.124
1.000	.025	.028	.107	.077
1.500	.021	.025	.000	.029
2.000	.021	.020	.044	.043
2.475	.000	.028	.000	.020

HU= 7.700 HD= 2.295 Q= .030 T=24.00

R	U1N/UO	U2N/UO	U3N/UO	U4N/UO
.000	1.000	.628	.285	.130
.166		.549	.277	
.333		.351	.252	
.500	.033	.177	.213	.115
1.000	.023	.027	.099	.076
1.500	.023	.022	-.000	.045
2.000	.023	.022	.034	.039
2.475	.023	.035	.015	.022

HU= 7.820 HD= 2.289 Q= .024 T=25.10

R	U1N/UO	U2N/UO	U3N/UO	U4N/UO
.000	1.000	.699	.313	.163
.166		.523	.296	
.333		.277	.280	
.500	.057	.164	.231	.123
1.000	.049	.028	.095	.076
1.500	.040	.000	.033	.028
2.000	.049	.028	.033	.039
2.475	.040	.044	.000	.000

HU= 7.900 HD= 2.283 Q= .020 T=26.00

R	U1N/U0	U2N/U0	U3N/U0	U4N/U0
.000	1.000	.687	.351	.165
.166		.464		
.333		.316		
.500	.042	.149	.268	.134
1.000	.042	.033	.113	.057
1.500	.024	.033	.032	.024
2.000	.042	.033	.000	.033
2.475	.024	.000	.000	.000

HU= 8.000 HD= 2.275 Q= .014 T=26.50

R	U1N/U0	U2N/U0	U3N/U0	U4N/U0
.000	1.000	.667	.331	.149
.166		.579		
.333		.340		
.500	.034	.210	.270	.153
1.000	.000	.066	.183	.088
1.500	.000	.033	.072	.047
2.000	.000	.033	.046	.034
2.475	.000	.000	.000	.034

THE CALCULATION OF UMN/U0 (SERIES 2)

HU=11.800 HD= 2.523 Q= .283 T=17.50

R	U1N/U0	U2N/U0	U3N/U0	U4N/U0
.000	1.000	.653	.382	.227
.166		.373	.300	.206
.333		.136	.188	.129
.500	.016	.010	.164	.115
1.000	.012	.006	.004	.040
1.500	.012	.000	-.009	-.006
2.000	.004	-.006	-.014	-.025
2.475	-.002	-.025	-.041	-.041

HU=11.960 HD= 2.519 Q= .277 T=17.50

R	U1N/U0	U2N/U0	U3N/U0	U4N/U0
.000	1.000	.663	.371	.220
.166		.325	.327	.197
.333		.099	.220	.162
.500	.024	.012	.133	.109
1.000	.018	.004	.000	.025
1.500	.015	.000	-.006	-.003
2.000	.009	-.005	-.011	-.019
2.475	.000	-.030	-.044	-.051

HU=12.250 HD= 2.525 Q= .285 T=17.50

R	U1N/U0	U2N/U0	U3N/U0	U4N/U0
.000	1.000	.623	.398	.236
.166		.293	.343	.208
.333		.075	.264	.179
.500	.018	.019	.121	.145
1.000	.019	.004	.003	.027
1.500	.018	.002	-.011	-.003
2.000	.015	.000	-.015	-.013
2.475	.010	-.026	-.052	-.047

HU=13.000 HD= 2.547 Q= .319 T=17.50

R	U1N/U0	U2N/U0	U3N/U0	U4N/U0
.000	1.000	.611	.384	.244
.166		.309	.317	.218
.333		.053	.204	.161
.500	.020	.024	.133	.123
1.000	.026	.006	.004	.033
1.500	.020	.004	-.005	.006
2.000	.016	.001	-.005	-.000
2.475	.011	-.041	-.072	-.068

HU=13.450 HD= 2.557 Q= .333 T=17.50

R	U1N/U0	U2N/U0	U3N/U0	U4N/U0
.000	1.000	.576	.367	.203
.166		.282	.269	.160
.333		.111	.137	.134
.500	.035	.040	.050	.108
1.000	.037	.015	.005	.047
1.500	.037	.010	.000	.034
2.000	.030	.005	.000	.013
2.475	.011	-.057	-.104	-.087

THE CALCULATION OF UMN/UO (SERIES 3)

HU=14.870 HD= 2.620 Q= .438 T=19.80

R	U1N/U0	U2N/U0	U3N/U0	U4N/U0
.000	1.000	.694	.369	.235
.166		.328	.300	.190
.333		.069	.179	.150
.500	.017	.014	.135	.087
1.000	.014	.009	-.010	.003
1.500	.014	.008	-.010	-.008
2.000	.009	.002	-.013	-.013
2.475	-.004	-.041	-.051	-.054

HU=14.620 HD= 2.611 Q= .422 T=20.00

R	U1N/U0	U2N/U0	U3N/U0	U4N/U0
.000	1.000	.687	.365	.232
.166		.374	.299	.200
.333		.056	.161	.154
.500	.020	.005	.078	.101
1.000	.016	.001	-.010	.006
1.500	.014	.000	-.011	-.006
2.000	.011	-.001	-.015	-.014
2.475	.005	-.030	-.061	-.070

HU=14.790 HD= 2.610 Q= .420 T=20.40

R	U1N/U0	U2N/U0	U3N/U0	U4N/U0
.000	1.000	.676	.364	.220
.166		.348	.289	.187
.333		.070	.165	.126
.500	.025	.007	.069	.095
1.000	.021	.006	-.010	.010
1.500	.020	.004	-.010	.002
2.000	.016	-.000	-.015	-.014
2.475	.001	-.028	-.073	-.067

HU=14.860 HD= 2.605 Q= .415 T=20.40

R	U1N/U0	U2N/U0	U3N/U0	U4N/U0
.000	1.000	.647	.362	.212
.166		.310	.292	.187
.333		.060	.166	.137
.500	.028	.007	.067	.088
1.000	.025	.004	-.007	.011
1.500	.018	.003	-.010	.001
2.000	.014	-.002	-.020	-.003
2.475	-.000	-.050	-.083	-.059

HU=14.980 HD= 2.607 Q= .416 T=21.00

R	U1N/U0	U2N/U0	U3N/U0	U4N/U0
.000	1.000	.646	.349	.173
.166		.253	.208	.147
.333		.031	.140	.098
.500	.033	.009	.023	.057
1.000	.027	.002	-.017	.012
1.500	.023	.000	-.018	.005
2.000	.015	-.009	-.022	-.012
2.475	-.007	-.070	-.126	-.110

HU=15.020 HD= 2.604 Q= .412 T=21.00

R	U1N/U0	U2N/U0	U3N/U0	U4N/U0
.000	1.000	.630	.263	.156
.166		.182	.189	.111
.333		.020	.053	.080
.500	.060	.012	.010	.040
1.000	.055	.010	-.017	.025
1.500	.037	.004	-.024	.025
2.000	.031	-.014	-.029	.009
2.475	-.012	-.138	-.217	-.055

THE CALCULATION OF UMN/U0 (SERIES 4)

HU=22.600 HD= 2.777 Q= .728 T=21.00

R	U1N/U0	U2N/U0	U3N/U0	U4N/U0
.000	1.000	.678	.326	.187
.166		.554	.308	.182
.333		.168	.218	.154
.500	.006	.025	.106	.095
1.000	.024	.012	.000	.031
1.500	.024	.004	-.013	.001
2.000	.005	-.009	-.044	-.063
2.475	-.031	-.139	-.199	-.190

HU=22.690 HD= 2.775 Q= .725 T=21.00

R	U1N/U0	U2N/U0	U3N/U0	U4N/U0
.000	1.000	.626	.300	.182
.166		.339	.258	.159
.333		.071	.189	.128
.500	.035	.026	.078	.094
1.000	.027	.011	.000	.039
1.500	.021	.009	-.023	-.692
1.500	.021	.009	-.022	.034
2.000	.005	-.018	-.053	-.061
2.475	-.033	-.153	-.229	-.225

HU=22.680 HD= 2.772 Q= .721 T=21.00

R	U1N/U0	U2N/U0	U3N/U0	U4N/U0
.000	1.000	.583	.260	.141
.166		.226	.232	.142
.333		.109	.109	.114
.500	.014	.018	.055	.096
1.000	.033	.020	-.013	.032
1.500	.040	.016	-.026	.019
2.000	-.005	-.038	-.080	-.052
2.475	-.078	-.184	-.358	-.359

HU=22.700 HD= 2.770 Q= .715 T=21.00

R	U1N/U0	U2N/U0	U3N/U0	U4N/U0
.000	1.000	.487	.129	.109
.166		.104	.089	.099
.333		-.027	.018	.088
.500	.041	.011	-.031	.077
1.000	.073	.032	-.035	.055
1.500	.078	.026	-.054	.021
2.000	.021	-.064	-.135	-.302
2.475	-.191	-.570	-.756	-.758

INVESTIGATIONS OF THE AXISYMMETRICAL CONFINED JET

by

Wen-shiuh Hwang

B.S., National Taiwan University, Formosa, 1963

AN ABSTRACT OF A THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1967

The difference in velocity between a jet and the region into which it is discharged will give rise to a pronounced degree of instability, the kinetic energy of the oncoming flow steadily being converted into kinetic energy of turbulence and the later steadily decaying through viscous shear.

The purpose of this thesis is to observe the above phenomena in a confined jet. The study of this type of jet has been based on the condition of continuity rather than the constancy of momentum flux and has been analysed by the introduction of some dimensionless groups.

For a confined jet without a secondary flow, there will be recirculation occurring within the region outside of the jet. For the case of a jet with a moving secondary flow, the recirculation will disappear. The velocity in the region outside the diffusion zone will be reduced by some amount which is dependent on the magnitude of the difference of the velocities of the two streams.

The experimental results obtained show that the process of mixing will be completed within a determinable distance downstream of the jet. The maximum velocity along the central axis of the jet will approach a constant value as the flow continues downstream.

This investigation indicates that the Gaussian function method used in the study of a free jet is inadequate for the treatment of the confined jet. However, the introduction of dimensionless groups provides a very good solution.